Automated Verification of Cyber-Physical Systems

A.A. 2022/2023
Corso di Laurea Magistrale in Informatica

Basic Notions

Igor Melatti

Università degli Studi dell'Aquila

Dipartimento di Ingegneria e Scienze dell'Informazione e Matematica

General Info for This Class

- Automated Verification of Cyber-Physical Systems is an elective course for the Master Degree in Computer Science
- Lecturer: Igor Melatti
- Where to find these slides and more:
  - `https://igormelatti.github.io/aut_ver_cps/20222023/index_eng.html`
  - also on MS Teams: "DT0759: Automated Verification of Cyber-Physical Systems (2022/23)", code **11xu0gi**
- 2 classes every week, 2 hours per class

Rules for Exams

- Each exam has a written part (50% of mark) and a project/paper (50% of mark)
  - each student may choose if making a project or reviewing a paper
  - teams of at most 2 students are allowed for projects
- Written exam will be a mix of open and closed questions on the whole exam program
- Project/paper may be discussed only after having passed the written exam
  - however, pre-evaluation is possible

Rules for Exams

- Project: perform verification of a given cyber-physical system
  - each team may choose one among the ones selected by lecturer
  - or may propose one (but wait for lecturer approval!)
  - each team will have to discuss its project with slides
- Paper: read a conference or journal paper and present it with slides
  - each student may choose one among the ones selected by lecturer
  - or may propose one (but wait for lecturer approval!)

Model Checking Problem

- Input: a system $\mathcal{S}$ and (at least) a property φ
  - more precisely, a *model* of $\mathcal{S}$ must be provided
  - that is, $\mathcal{S}$ must be described in some suitable language
- Output:

  PASS $\mathcal{S}$ satisfies φ, i.e., $\mathcal{S} \models \varphi$
  - the system $\mathcal{S}$ is correct w.r.t. the property φ
  - mathematical certification, much better than, e.g., testing

  FAIL $\mathcal{S}$ does not satisfy φ, i.e., $\mathcal{S} \not\models \varphi$
  - the system $\mathcal{S}$ is buggy w.r.t. the property φ
  - a *counterexample* providing evidence of the error is also returned

Model Checking vs. Other Verification Techniques

- Model checking is fully automatic
  - a model checker only needs the description of $\mathcal{S}$ and the property φ
  - "press button and go"
  - this is not true for other verification tools such as proof checkers, which require human intervention in the process
- Model checking is correct for both PASS and FAIL
  - unless the description of $\mathcal{S}$, or the property φ, are wrong
  - this is not true for other verification techniques such as testing, which only guarantees the FAIL result
  - a buggy system may pass all tests, because the error is in some *corner case*

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Model Checking Shortcomings

- Only works for finite-state systems
  - typical example: you may verify a system with 3, 4 or 5 processes, but not with n processes, for a generic n
- Requires skilled personnel to write descriptions (and properties)
  - must know both the model checker language and the system
  - however, less skilled than a proof checker user
  - very few exceptions in which the model is automatically extracted from the system
  - also direct translations from digital circuits to NuSMV are available
- Very resource demanding
  - besides PASS and FAIL, also OutOfMem and OutOfTime are expected results...
  - bounded model checking: PASS is limited to execution up to a given number of steps

Model Checking Algorithms

Two main categories:

Explicit visit the graph induced by the description of $\mathcal{S}$

- very good for invariants and LTL model checking of communication protocols
- on-the-fly generation of the graph: only the reachable states are stored, the adjacency matrix is implicitly given by the description of $\mathcal{S}$
- Murphi, SPIN

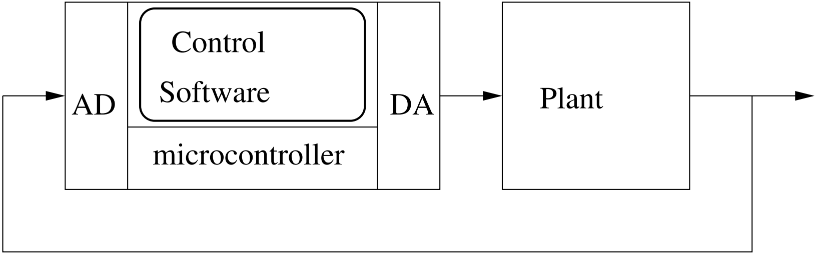
Symbolic represent sets of states and transition relations as OBDDs

- very good for LTL and CTL model checking of hardware-like systems
- all translated into a boolean formula
- also SAT tools may be used (bounded model checking)

Cyber-Physical Systems

- A Cyber-Physical System (CPS) is a system where a physical system is controlled and/or monitored by a software
- They are either partially or fully autonomous
  - we will mainly deal with fully autonomous CPSs
- Examples are everywhere:
  - Internet of Things devices
  - Unmanned Autonomous Vehicles
  - Drones
  - Medical Devices
  - Embedded Systems
  - ...

Cyber-Physical Systems with Controllers



CPSs with Controllers: Classical Examples

Buck DC/DC Converter

CPSs with Controllers: Classical Examples

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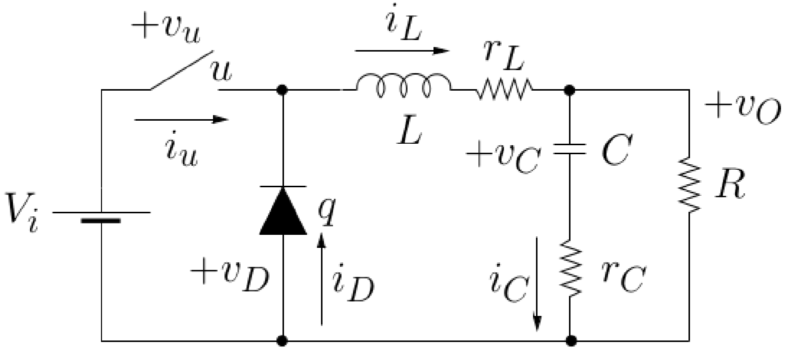

Continuous time dynamics

$$\dot{i}_L = a_{1,1} i_L + a_{1,2} v_O + a_{1,3} v_D \quad (1)$$
$$\dot{v}_O = a_{2,1} i_L + a_{2,2} v_O + a_{2,3} v_D \quad (2)$$

$$q \rightarrow v_D = R_{\mathrm{on}} i_D \quad (3) \qquad \bar{q} \rightarrow v_D = R_{\mathrm{off}} i_D \quad (7)$$
$$q \rightarrow i_D \geq 0 \quad (4) \qquad \bar{q} \rightarrow v_D \leq 0 \quad (8)$$
$$u \rightarrow v_u = R_{\mathrm{on}} i_u \quad (5) \qquad \bar{u} \rightarrow v_u = R_{\mathrm{off}} i_u \quad (9)$$
$$v_D = v_u - V_{in} \quad (6) \qquad i_D = i_L - i_u \quad (10)$$

where:

- i_L, v_O are state variables
- $u \in \{0, 1\}$ is the action

Discrete time dynamics with sampling time T

$$
\begin{aligned}
{i_L}' &= (1 + Ta_{1,1})i_L + Ta_{1,2}v_O + Ta_{1,3}v_D && (11)\\
{v_O}' &= Ta_{2,1}i_L + (1 + Ta_{2,2})v_O + Ta_{2,3}v_D. && (12)
\end{aligned}
$$

$$
\begin{aligned}
q &\rightarrow v_D = R_{\mathrm{on}}\, i_D && (13) \qquad & \bar{q} &\rightarrow v_D = R_{\mathrm{off}}\, i_D && (17)\\
q &\rightarrow i_D \geq 0 && (14) \qquad & \bar{q} &\rightarrow v_D \leq 0 && (18)\\
u &\rightarrow v_u = R_{\mathrm{on}}\, i_u && (15) \qquad & \bar{u} &\rightarrow v_u = R_{\mathrm{off}}\, i_u && (19)\\
v_D &= v_u - V_{in} && (16) \qquad & i_D &= i_L - i_u && (20)
\end{aligned}
$$

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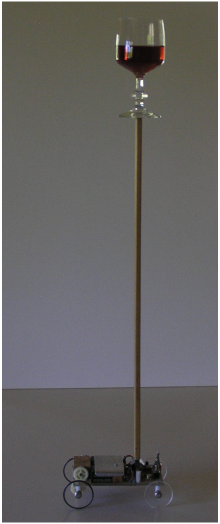
CPSs with Controllers: Classical Examples

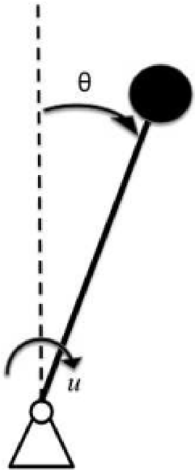
- Goal: keep v_O in a desired safe interval
  - typically, $5 - 0.01V \leq v_O \geq 5 + 0.01V$
- Notwithstanding the input voltage V_i and the resistance R may vary in some given interval
  - typically, $R = 5 \pm 25\%\Omega$, $V_i = 15 \pm 25\%V$
- Effectively used in laptops: from battery voltage (V_i) to laptop processor voltage (v_O)

CPSs with Controllers: Classical Examples

Inverted Pendulum



Inverted Pendulum



CPSs with Controllers: Classical Examples

Continuous time dynamics

$$\ddot{\theta} = \frac{g}{l}\sin\theta + \frac{1}{ml^2}Fu$$

where:

- θ is the state variable
- $u \in \{0, 1\}$ is the action
- m, l, F are system parameters

Continuous time dynamics

$$\dot{x}_1 = x_2 \tag{21}$$

$$\dot{x}_2 = \frac{g}{l}\sin x_1 + \frac{1}{ml^2}Fu \tag{22}$$

Discrete time dynamics with sampling time T

$$x_1' = x_1 + Tx_2 \tag{23}$$

$$x_2' = x_2 + T\frac{g}{l}\sin x_1 + T\frac{1}{ml^2}Fu \tag{24}$$

In This Course

To deal with cyber-physical systems:

- Probabilistic Model Checking
  - rather than "are there errors?", it is "is the error probability low enough?"
  - the system is probabilistic, i.e., a Markov Chain
- System Level Formal Verification
  - directly use a simulator instead of describing the system within the model checker
  - this will also need some background on systems simulation

In This Course

To deal with cyber-physical systems:

- Statistical Model Checking
  - rather than "are there errors?", it is "is the error probability low enough?"
  - the system is a non-probabilistic simulator
  - the answer is given with some statistical confidence
- Automatic Synthesis of Controllers
  - rather than "are there errors in this system?", it is "generate a controller so that errors are avoided"

Formal Verification Methodologies: a Classification

There are two macro-categories:

- *Interactive methods*
  - as the name suggests, not (fully) automatic
  - human intervention is typically required
  - in this course, we do not deal with such techniques
- *Automatic methods*
  - only human intervention is to *model* the system

Formal Verification Methodologies: a Classification

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- *Automatic methods*
  - only human intervention is to *model* the system
- There also exist hybridations among the two categories

Interactive Methods

- Also called *proof checkers*, *proof assistants* or *high-order theorem provers*
- Tools which helps in building a mathematical proof of correctness for the given system and property
- Pros
  - virtually no limitation to the type of system and property to be verified
- Cons
  - highly skilled personnel is needed
  - both in mathematical logic and in deductive reasoning
  - needed to “help” tools in building the proof

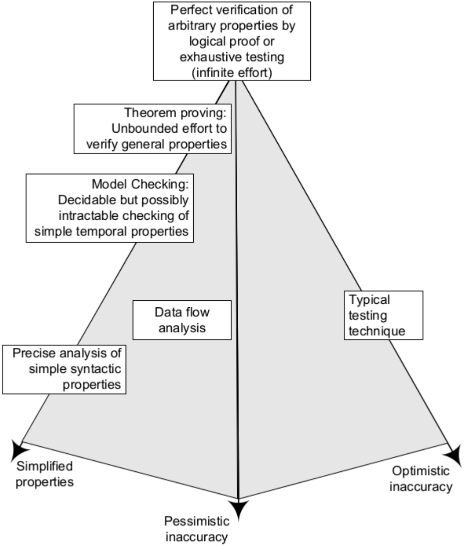
Interactive Methods

- Used for projects with high budgets
- For which the automatic methods limitations are not acceptable
  - used, e.g., to prove correctness of microprocessor circuits or OS microkernels
- Some tools in this category (see `https://en.wikipedia.org/wiki/Proof_assistant`):
  - HOL
  - PVS
  - Coq

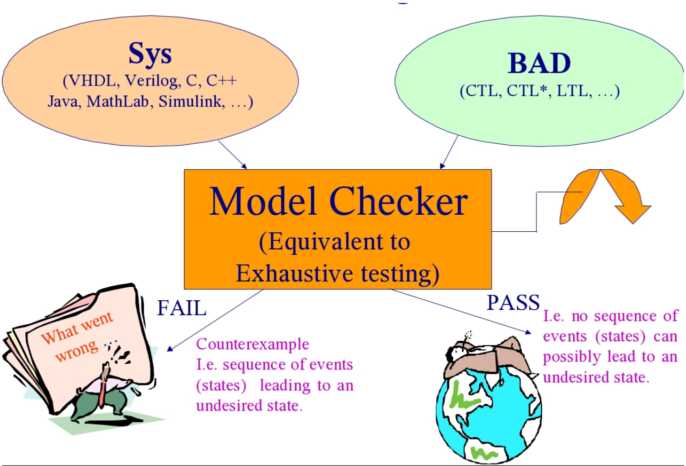
Automatic Methods

- Commonly dubbed *Model Checking*
- Model Checking software tools are called *model checkers*
- There are some tens model checkers developed; the most important ones are listed in `https://en.wikipedia.org/wiki/List_of_model_checking_tools`
- Many are freely downloadable and modifiable for research and study purposes
- Research area with many achievements in over 30 years

Verification Tradeoffs



The Model Checking Dream



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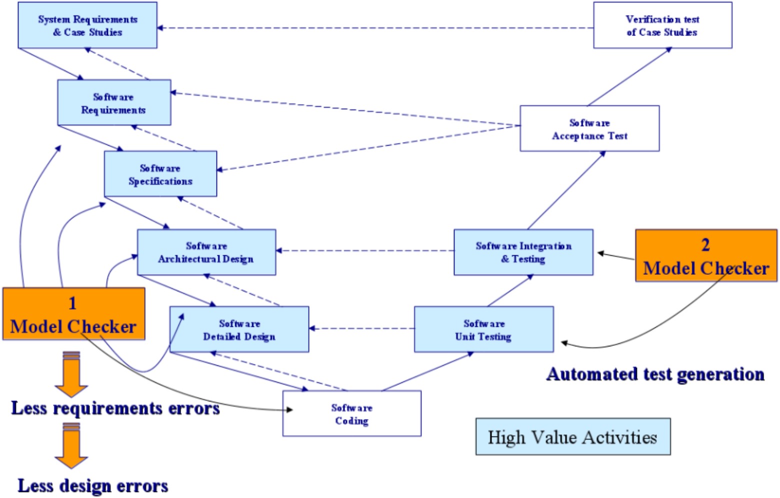


Actual Model Checking

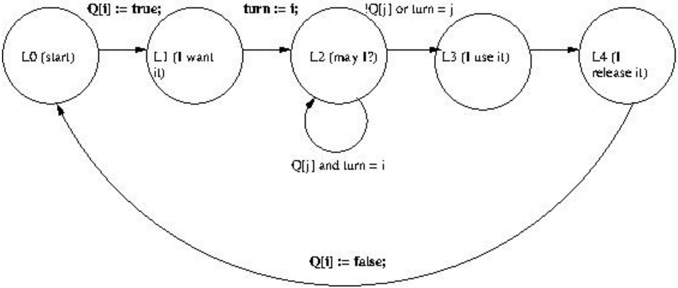
- In order to have this computationally feasible, we need a strong assumption on the system under verification (SUV)
- I.e., it must have a *finite number of states*
  - *Finite State System* (FSS)
- In this way, model checkers "simply" have to implement reachability-related algorithms on graphs
- Such finite state assumption, though strong, is applicable to many interesting systems
  - that is: many systems are actually FSSs
  - or they may be approximated as such
  - or a part of them may be approximated as such

What Is a *State*?

- There are many notions of "state" in computer science
- Model checking states are *not* the ones in UML-like state diagrams
- Model checking states are similar to operational semantics states
- That is: suppose that a system is "described" by n variables
- Then, a state is an assignment to all n variables
  - given $D_1, \ldots, D_n$ as our n variables domains, then a state is $s \in \times_{i=1}^{n} D_i$

What Is a *State*: Example

- We have two identical processes accessing to a shared resource
  - in the figure below, i, j denote the two processes
  - the well-known Peterson algorithm is used

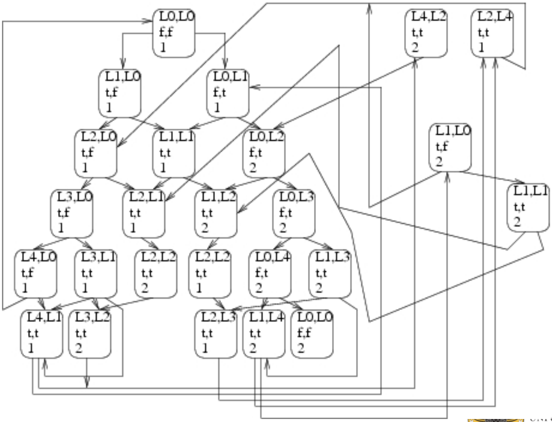


What Is a *State*: Example

- The 5 “states” in the preceding figure are actually *modalities*
- From a model checking point of view, they correspond to *multiple* states
- To see which are the actual states, let us model this system with the following variables:
    - m_i, with $i = 1, 2$: the modality for process i
    - Q_i, with $i = 1, 2$: Q_i is a boolean which holds iff process i wants to access the shared resource
    - `turn`: shared variable

What Is a *State*: Example

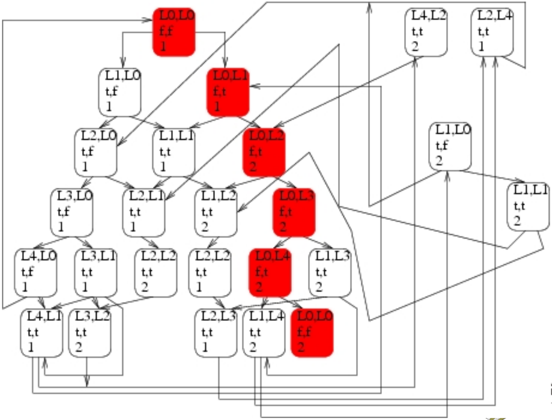
- Thus, the resulting model checking states are the following:



What Is a *State*: Example

- There are 25 *reachable states*
  - assuming state $\langle L0, L0, f, f, 1\rangle$ as the starting one
- All *possible* states are 200
  - there are 3 variables with two possible values (the 2 variables `Q`, plus the `turn` variable) and 2 variables (`P`) with 5 possible values, thus $2^3 \times 5^2$ overall assignments
- The `L0` modality for the first process encloses 6 (reachable) states

What Is a *State*: Example



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- The `L0` modality for the first process encloses 6 (reachable) states
- No need of guards on transitions!
  - guards will be needed for systems with external inputs

From State Diagrams to Model Checking

- The UML-like state diagram is often useful to write the model
  - as we will see, this will depend on the model checker *input language*
- It is the model checker task to extract the global (reachable) graph as seen before
- And then analyze it

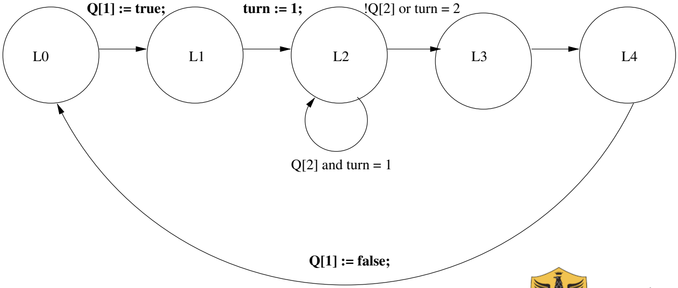
- Example: G. L. Peterson protocol for mutual exclusion of 2 processes (1981)

Peterson's Algorithm

```
boolean flag [2];
int turn;
void P0()
{
     while (true) {
          flag [0] = true;
          turn = 1;
          while (flag [1] && turn == 1) /* do nothing */;
          /* critical section */;
          flag [0] = false;
          /* remainder */;
     }
}
void P1()
{
     while (true) {
          flag [1] = true;
          turn = 0;
          while (flag [0] && turn == 0) /* do nothing */;
          /* critical section */;
          flag [1] = false;
          /* remainder */
     }
}
void main()
{
     flag [0] = false;
     flag [1] = false;
     parbegin (P0, P1);
}
```

Murphi

- Example: G. L. Peterson protocol for mutual exclusion of 2 processes (1981)
- UML-like state diagram: this is the first process; the second may be obtained exchanging 1's with 2's and viceversa



Murphi

- Example: G. L. Peterson protocol for mutual exclusion of 2 processes (1981)
  - two identical processes
  - each applies Peterson protocol to access to the critical section L3
  - the first issuing the request enters L3
  - `Q` is a global variable, defined as an array of two integers
    - each process i may modify `Q`[i] and read `Q`[$(i+1) \mod 2$]
  - `turn` is another global variable, which may be both read and modified by both processes

Murphi

- Murphi description for Peterson protocol: let's start with the variables
  - of course `turn` and `Q`, but also two variables `P` for the modality ("states" in the UML-like state diagram)
  - see `01.2_peterson.no_rulesets.no_parametric.m`
  - to this aim, we define constants and types
  - the `N` constant (number of processes) is here fictious: only 2 processes, not more
  - this version of Peterson protocol only works for 2 processes
- thus, the state space is
  $S = \texttt{label\_t}^2 \times \{\texttt{true}, \texttt{false}\}^2 \times \{1, 2\}$

Variables for Murphi Model Describing Peterson Protocol

P	$v \in \{L0, L1, L2, L3, L4\}$	$v \in \{L0, L1, L2, L3, L4\}$
Q	$v \in \{true, false\}$	$v \in \{true, false\}$
turn	$v \in \{1..N\}$	

Murphi

- Hence, $|S| = 5^2 \times 2^2 \times 2 = 200$ (there are 200 possible states)
  - as a matter of comparison, the "state" L0 in the UML-like state diagram actually contains $5^1 \times 2^2 \times 2 = 40$ states...
- However, as we will see, *reachable* states are about 10 times less
- 2 initial states: `turn` may be initialied with any value in its domain
- Note that `01.2_peterson.no_rulesets.no_parametric.m` we have rules repeated 2 times in a nearly equal fashion
- This can be done in this very simple model, but in general descriptions must be *parametric*

Murphi

- If we want to check Peterson with 3 processi, currently we would have to add one more rule in the descriprion
- Instead, it must be possible to only change the value of `N` from 2 to 3
- To write parametric descriptions in Murphi, rules are grouped with *rulesets*
  - an index will allow to describe the behavior of the generic process *i*
  - see `02.2_peterson.with_rulesets.no_parametric.m`

Murphi

- Invariant: of course, at any execution instant, there must be only one state in L3 (mutual exclusion)
- In a first order logic, it would be something like:

$$\forall k \in \{1, \ldots, \mathtt{N}\}.\ \forall k' \in \{1, \ldots, \mathtt{N}\}.\ (k \neq k' \wedge \mathtt{P}[k] = L3) \Rightarrow \mathtt{P}[k'] \neq L3$$

- Or, as a reverse:

$$\neg(\exists k \in \{1, \ldots, \mathtt{N}\}.\ \exists k' \in \{1, \ldots, \mathtt{N}\}.\ k \neq k' \wedge \mathtt{P}[k] = L3 \wedge \mathtt{P}[k'] = L3)$$

- In the first version, it is stated what is correct to happen
- In the first version, it is stated what is wrong to happen
- In both `00.2_peterson.with_rulesets.no_parametric.m` and `02.2_peterson.no_rulesets.no_parametric.m` invariant is not parametric
- See `03.2_peterson.with_rulesets.parametric.m`

Kripke Structures

- Let AP be a set of "atomic propositions"
  - in the sense of first-order logic: each atomic proposition is either true or false
  - tipically identified with lower case letters $p, q, \ldots$
- A *Kripke Structure* (KS) over AP is a 4-tuple $\langle S, I, R, L \rangle$
  - S is a finite set, its elements are called *states*
  - $I \subseteq S$ is a set of *initial states*
  - $R \subseteq S \times S$ is a *transition relation*
  - $L : S \to 2^{AP}$ is a *labeling function*

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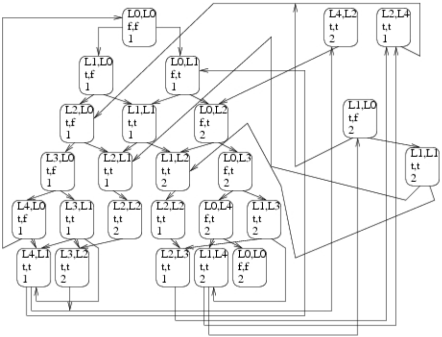
Labeled Transition Systems

- A *Labeled Transition System* (LTS) is a 4-tuple $\langle S, I, \Lambda, \delta \rangle$
  - S is a finite set of states as before
  - $I \subseteq S$ is a set of initial states as before (not always included)
  - Λ is a finite set of *labels*
  - $\delta \subseteq S \times \Lambda \times S$ is a *labeled transition relation*

Peterson's Mutual Exclusion as a Kripke Structure

- $S = \{(p_1, p_2, q_1, q_2, t) \mid p_1, p_2 \in \{\mathrm{L0}, \mathrm{L1}, \mathrm{L2}, \mathrm{L3}, \mathrm{L4}\}, q_1, q_2 \in \{0, 1\}, t \in \{1, 2\}\} = \{\mathrm{L0}, \mathrm{L1}, \mathrm{L2}, \mathrm{L3}, \mathrm{L4}\}^2 \times \{0, 1\}^2 \times \{1, 2\}$
- $I = \{\mathrm{L0}\}^2 \times \{0\}^2 \times \{1, 2\}$
- R: see next slide
- $AP = \{(\mathtt{P}_1 = v) \mid v \in \{\mathrm{L0}, \mathrm{L1}, \mathrm{L2}, \mathrm{L3}, \mathrm{L4}\}\} \cup \{(\mathtt{P}_2 = v) \mid v \in \{\mathrm{L0}, \mathrm{L1}, \mathrm{L2}, \mathrm{L3}, \mathrm{L4}\}\} \cup \{(\mathtt{Q}_1 = v) \mid v \in \{0, 1\}\} \cup \{(\mathtt{Q}_2 = v) \mid v \in \{0, 1\}\} \cup \{(\mathtt{turn} = v) \mid v \in \{1, 2\}\}$
  - e.g.: $L(\mathrm{L0}, \mathrm{L0}, 0, 0, 1)) = \{(\mathtt{P}_1 = \mathrm{L0}), (\mathtt{P}_2 = \mathrm{L0}), (\mathtt{Q}_1 = 0), (\mathtt{Q}_2 = 0), (\mathtt{turn} = 1)\}$

Peterson's Mutual Exclusion as a Kripke Structure



E.g.: $((\mathrm{L}0, \mathrm{L}0, 0, 0, 1), (\mathrm{L}1, \mathrm{L}0, 1, 0, 1)) \in R$, whilst
$((\mathrm{L}0, \mathrm{L}0, 0, 0, 1), (\mathrm{L}2, \mathrm{L}0, 0, 0, 1)) \notin R$
Of course, $|R|$ = number of arrows in figure above

Kripke Structure vs Labeled Transition Systems

- KSs have atomic propositions on states, LTSs have labels on transitions
- In model checking, atomic propositions are mandatory
  - to specify the formula to be verified, as we will see
  - a first example was the invariant in Murphi
- Instead, it is not required to have a label on transitions
  - Murphi allows to do so, but it is optional
  - may be easily added automatically, if needed
- Labels are typically needed when:
  - we deal with macrostates, as in UML state diagrams
  - when we are describing a complex system by specifying its sub-components, so labels are used for synchronization

Total Transition Relation

- In many cases, the transition relation R is required to be *total*
- $\forall s \in S . \exists s' \in S : (s, s') \in R$
  - this of course allows also $s = s'$ (*self loop*)
- In the Peterson's example, the relation is actually total
  - Murphi allows also non-total relations, by using option `-ndl`
  - note however that not giving option `-ndl` is stronger: $\forall s \in S . \exists s' \in S : s \neq s' \wedge (s, s') \in R$
  - otherwise, if s is s.t. $\forall s' .\ s = s' \vee (s, s') \notin R$, Murphi calls s a *deadlock* state
  - that is, you cannot go anywhere, except possibly self looping on s
- By deleting any rule, we will obtain a non-total transition relation

Non-Determinism

- The transition relation is, as the name suggests, a relation
- Thus, starting from a given state, it is possible to go to many different states
  - in a deterministic system, $\forall s_1, s_2, s_3 \in S.\ (s_1, s_2) \in R \land (s_1, s_3) \in R \to s_2 = s_3$
  - this does not hold for KSs
- This means that, starting from state s_1, the system may *non-deterministically* go either to s_2 or to s_3
  - or many other states
- Motivations for non-determinism: modeling choices!
  - underspecified subsystems
  - unpredictable interleaving
  - interactions with an uncontrollable environment
  - ...

Some Useful Notation

- Given a KS $\mathcal{S} = \langle S, I, R, L \rangle$, we can define:
  - the *predecessor* function $\mathrm{Pre}_{\mathcal{S}} : S \to 2^S$
    - defined as $\mathrm{Pre}_{\mathcal{S}}(s) = \{s' \in S \mid (s', s) \in R\}$
    - we will write simply $\mathrm{Pre}(s)$ when $\mathcal{S}$ is understood
  - the *successor* function $\mathrm{Post} : S \to 2^S$
    - defined as $\mathrm{Post}(s) = \{s' \in S \mid (s, s') \in R\}$
- Note that, if $\mathcal{S}$ is deterministic, $\forall s \in S.\ |\mathrm{Post}(s)| \leq 1$

Paths in KSs

- A path (or *execution*) on a KS $\mathcal{S} = \langle S, I, R, L \rangle$ is a sequence $\pi = s_0 s_1 s_2 \ldots$ such that:
  - $\forall i \geq 0.\ s_i \in S$ (it is composed by states)
  - $\forall i \geq 0.\ (s_i, s_{i+1}) \in R$ (it only uses valid transitions)
- We will denote i-th state of a path as $\pi(i) = s_i$
- Note that paths in LTSs also have actions: $\pi = s_0 a_0 s_1 a_1 \ldots$ s.t. $(s_i, a_i, s_{i+1} \in \delta)$

Paths in KSs

- The *length* of a path π is the number of states in π
  - paths can be either finite $\pi = s_0 s_1 \ldots s_n$, in which case $|\pi| = n + 1$
  - or infinite $\pi = s_0 s_1 \ldots$, in which case $|\pi| = \infty$
- We will denote the prefix of a path up to i as $\pi|_i = s_0 \ldots s_i$
  - a prefix of a path is always a finite path
- A path π is *maximal* iff one of the following holds
  - $|\pi| = \infty$
  - $|\pi| = n + 1$ and $|\mathrm{Post}(\pi(n))| = 0$
    - that is, $\forall s \in S.\ (\pi(n), s) \notin R$
    - i.e., the last state of the path has no successors
    - often called *terminal state*
- If R is total, maximal paths are always infinite
  - for many model checking algorithms, this is required

Reachability

- The set of paths of $\mathcal{S}$ starting from $s \in S$ is denoted by $\mathrm{Path}(\mathcal{S}, s) = \{\pi \mid \pi \text{ is a path in } \mathcal{S} \land \pi(0) = s\}$
- The set of paths of $\mathcal{S}$ is denoted by $\mathrm{Path}(\mathcal{S}) = \cup_{s \in I} \mathrm{Path}(\mathcal{S}, s)$
  - that is, they must start from an initial state
- A state $s \in S$ is *reachable* iff $\exists \pi \in \mathrm{Path}(\mathcal{S}), k \leq |\pi| : \ \pi(k) = s$
  - i.e., there exists a path from an initial state leading to s through valid transitions
- The set of reachable states is defined by $\mathrm{Reach}(\mathcal{S}) = \{\pi(i) \mid \pi \in \mathrm{Path}(\mathcal{S}), i \leq |\pi|\}$

Safety Property Verification

- Verification of *invariants*: nothing bad happens
- The property is a formula $\varphi : S \to \{0, 1\}$
  - built using boolean combinations of atomic propositions in $p \in AP$
  - i.e., the syntax is

$$\Phi : (\Phi) \mid \Phi \wedge \Phi \mid \Phi \vee \Phi \mid \neg\Phi \mid p$$

- The KS $\mathcal{S}$ satisfies φ iff φ holds on all reachable states
  - $\forall s \in \mathrm{Reach}(\mathcal{S}).\ \varphi(s) = 1$
- Note that it may happen that $\varphi(s) = 0$ for some $s \in S$: never mind, if $s \notin \mathrm{Reach}(\mathcal{S})$

How to Verify a Murphi Description $\mathcal{M}$

- Theoretically, extract KS $\mathcal{S}$ and property φ from $\mathcal{M}$ as described above
  - for a given invariant I in $\mathcal{M}$, $\varphi(s) = \zeta(I, s)$ for all $s \in S$
- Then, KS $\mathcal{S}$ satisfies φ iff φ holds on all reachable states
  - $\forall s \in \mathrm{Reach}(\mathcal{S}).\ \varphi(s) = 1$
- Thus, consider KS as a graph and perform a visit
  - states are nodes, transitions are edges
- If a state e s.t. $\varphi(e) = 0$ is found, then we have an error
- Otherwise, all is ok

How to Verify a Murphi Description $\mathcal{M}$

- From a practical point of view, many optimization may be done, but let us stick to the previous scheme
- The worst case time complexity for a DFS or a BFS is $O(|V| + |E|)$ (and same for space complexity)
- For KSs, this means $O(|S| + |R|)$, thus it is linear in the size of the KS
- Is this good? NO! Because of the *state space explosion problem*
- Assuming that B bits are needed to encode each state
  - i.e., $B = \sum_{i=1}^{n} b_i$, being b_i the number of bits to encode domain D_i
- We have that $|S| = O(2^B)$

State Space Explosion

- The "practical" input dimension is B, rather than $|S|$ or $|R|$
- Typically, for a system with N components, we have $O(N)$ variables, thus $O(B)$ encoding bits
- It is very common to verify a system with N components, and then (if N is ok) also for $N + 1$ components
    - verifying a system with a generic number N of components is a typically proof checker task...
- This entails an esponential increase in the size of $|S|$
- Thus we need "clever" versions of BFS/DFS

Standard BFS: No Good for Model Checking

- Assumes that all graph nodes are in RAM
- For KSs, graph nodes are states, and we now there are too many
  - state space explosion
- You also need a full representation of the graph, thus also edges must be in RAM
  - using adjacency matrices or lists does not change much
  - for real-world systems, you may easily need TB of RAM
- Even if you have all the needed RAM, there is a huge preprocessing time needed to build the graph from the Murphi specification
- Then, also BFS itself may take a long time

Murphi BFS

- We need a definition inbetween the model and the KS: NFSS (Nondeterministic Finite State System)
- $\mathcal{N} = \langle S, I, \mathrm{Post} \rangle$, plus the invariant φ
  - S is the set of states, $I \subseteq S$ the set of initial states
  - $\mathrm{Post} : S \to 2^S$ is the successor function as defined before
    - given a state s, it returns T s.t. $t \in T \to (s, t) \in R$
  - no labeling, we already have φ

Murphi BFS

- KSs and NFSSs differ on having Post instead of R
- Post may easily be defined from the Murphi specification
- Such definition is implicit, as programming code, thus avoiding to store adjacency matrices or lists
  - $t \in \mathrm{Post}(s)$ iff there is a rule $T_i \in T$ s.t. T_i guard is true in s and T_i body changes s to t
    - see above for using η and ζ
  - Essentially, if the current state is s, it is sufficient to inspect all (flattened) rules in the Murphi specification $\mathcal{M}$
    - for all guards which are enabled in s, execute the body so as to obtain t, and add t to $\mathrm{next}(s)$
  - This is done "on the fly", only for those states s which must be explored

Murphi Simulation

```
void Make_a_run(NFSS 𝒩, invariant φ)
{
 let 𝒩 = ⟨S, I, Post⟩;
 s_curr = pick_a_state(I);
 if (!φ(s_curr))
  return with error message;
 while (1) { /* loop forever */
  s_next = pick_a_state(Post(s_curr));
  if (!φ(s_next))
   return with error message;
  s_curr = s_next;
 }
}
```

Murphi Simulation

```
void Make_a_run(NFSS 𝒩, invariant φ)
{
 let 𝒩 = ⟨S, I, Post⟩;
 s_curr = pick_a_state(I);
 if (!φ(s_curr))
  return with error message;
 while (1) { /* loop forever */
  if (Post(s_curr) = ∅)
   return with deadlock message;
  s_next = pick_a_state(Post(s_curr));
  if (!φ(s_next))
   return with error message;
  s_curr = s_next;
 }
}
```

Murphi Simulation

- Similar to testing
- If an error is found, the system is bugged
  - or the model is not faithful
  - actually, Murphi simulation is used to understand if the model itself contains errors
- If an error is not found, we cannot conclude anything
- The error state may lurk somewhere, out of reach for the random choice in `pick_a_state`

Standard BFS (Cormen-Leiserson-Rivest)

```
BFS(G, s)
for ogni vertice u ∈ V[G] − {s}
    do  color[u] ← WHITE
        d[u] ← ∞
        π[u] ← NIL
color[s] ← GRAY
d[s] ← 0
π[s] ← NIL
Q ← {s}
while Q ≠ ∅
    do  u ← head[Q]
        for ogni v ∈ Adj[u]
            do if color[v] = WHITE
                then  color[v] ← GRAY
                      d[v] ← d[u] + 1
                      π[v] ← u
                      ENQUEUE(Q, v)
        DEQUEUE(Q)
        color[u] ← BLACK
```

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Murphi BFS

```
FIFO_Queue Q;
HashTable T;

bool BFS(NFSS 𝒩, AP φ)
{
 let 𝒩 = (S, I, Post);
 foreach s in I {
  if (!φ(s))
   return false;
 }
 foreach s in I
  Enqueue(Q, s);
 foreach s in I
  HashInsert(T, s);
```

Murphi BFS

```
while (Q ≠ ∅) {
 s = Dequeue(Q);
 foreach s_next in Post(s) {
  if (!φ(s_next))
   return false;
  if (s_next is not in T) {
   Enqueue(Q, s_next);
   HashInsert(T, s_next);
  } /* if */ } /* foreach */ } /* while */
return true;
}
```

Murphi BFS

- Edges are never stored in memory
- (Reachable) states are stored in memory only at the end of the visit
  - inside hashtable `T`
- This is called *on-the-fly* verification
- States are marked as visited by putting them inside an hashtable
  - rather than coloring them as gray or black
  - which needs the graph to be already in memory

State Space Explosion

- State space explosion hits in the FIFO queue `Q` and in the hashtable `T`
  - and of course in running time...
- However, `Q` is not really a problem
  - it is accessed *sequentially*
  - always in the front for extraction, always in the rear for insertion
  - can be efficiently stored using disk, much more capable of RAM
- `T` is the real problem
  - random access, not suitable for a file
  - what to do?
  - before answering, let's have a look at Murphi code

Murphi Usage

- As for all *explicit* model checker, a Murphi verification has the following steps:
  0. compile Murph source code and write a Murphi model `model.m`
  1. invoke Murphi compiler on `model.m`: this generates a file `model.cpp`
     - `mu options model.m`
     - see `mu -h` for available options
  2. invoke C++ compiler on `model.cpp`: this generates an executable file
     - `g++ -Ipath_to_include model.cpp -o model`
     - `path_to_include` is the `include` directory inside Murphi distribution
  3. invoke the executable file
     - `./model options`
     - see `./model -h` for available options

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LTL Syntax

$$\Phi ::= p \mid \Phi_1 \wedge \Phi_2 \mid \neg\Phi \mid (\Phi) \mid \mathbf{X}\Phi \mid \Phi_1 \mathbf{U} \Phi_2$$

- Other derived operators:
  - of course true, false, OR and other propositional logic connectors
  - future (or eventually): $\mathbf{F}\Phi = \text{true } \mathbf{U} \Phi$
  - globally: $\mathbf{G}\Phi = \neg(\text{true } \mathbf{U} \neg\Phi)$
  - release: $\Phi_1 \mathbf{R} \Phi_2 = \neg(\neg\Phi_1 \mathbf{U} \neg\Phi_2)$
  - weak until: $\Phi_1 \mathbf{W} \Phi_2 = (\Phi_1 \mathbf{U} \Phi_2) \vee \mathbf{G}\Phi_1$
- Other notations:
  - next: $\mathbf{X}\Phi = \bigcirc\Phi$
  - $\mathbf{G}\Phi = \Box\Phi$
  - $\mathbf{F}\Phi = \Diamond\Phi$
- We are dropping *past operators*, thus this is *pure future LTL*

LTL Semantics

- Goal: formally defining when $\mathcal{S} \models \varphi$, being $\mathcal{S}$ a KS and φ an LTL formula
  - we say that $\mathcal{S}$ *satisfies* φ, or φ *holds in* $\mathcal{S}$
- This is true when, for all paths π of $\mathcal{S}$, π satisfies φ
  - i.e., $\forall \pi \in \mathrm{Path}(\mathcal{S}).\ \pi \models \varphi$
  - symbol $\models$ is overloaded...
- For a given π, $\pi \models \varphi$ iff $\pi, 0 \models \varphi$
- Finally, to define when $\pi, i \models \varphi$, a recursive definition over the recursive syntax of LTL is provided
  - $\pi \in \mathrm{Path}(\mathcal{S}), i \in \mathbb{N}$

LTL Semantics for $\pi, i \models \varphi$

- $\forall \pi \in \mathrm{Path}(\mathcal{S}), i \in \mathbb{N}.\ \pi, i \models \mathrm{true}$
- $\pi, i \models p$ iff $p \in L(\pi(i))$
- $\pi, i \models \Phi_1 \wedge \Phi_2$ iff $\pi, i \models \Phi_1 \wedge \pi, i \models \Phi_2$
- $\pi, i \models \neg\Phi$ iff $\pi, i \not\models \Phi$
- $\pi, i \models \mathbf{X}\Phi$ iff $\pi, i+1 \models \Phi$
- $\pi, i \models \Phi_1 \mathbf{U} \Phi_2$ iff $\exists k \geq i :\ \pi, k \models \Phi_2 \wedge \forall i \leq j < k.\ \pi, j \models \Phi_1$

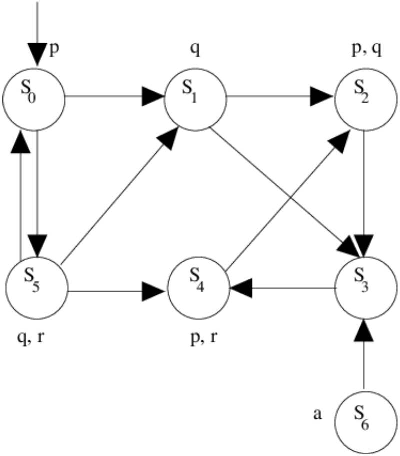
LTL Semantics for Added Operators

- It is easy to prove that:
  - $\pi, i \models \mathbf{G}\Phi$ iff $\forall j \geq i.\ \pi, j \models \Phi$
  - $\pi, i \models \mathbf{F}\Phi$ iff $\exists j \geq i.\ \pi, j \models \Phi$
  - $\pi, i \models \Phi_1\ \mathbf{R}\ \Phi_2$ iff $\forall j \geq i.\ (\forall k < j.\ \pi, k \models \Phi_1) \rightarrow \pi, j \models \Phi_2$
  - $\pi, i \models \Phi_1\ \mathbf{W}\ \Phi_2$ iff $(\forall j \geq i.\ \pi, j \models \Phi_1) \vee (\exists k \geq i:\ \pi, k \models \Phi_2 \wedge \forall i \leq j < k.\ \pi, j \models \Phi_1)$
- For many formulas, it is silently required that paths are infinite
- That's why transition relations in KSs must be total

Safety and Liveness Properties in LTL

- Given an LTL formula φ, φ is a safety formula iff $\forall \mathcal{S}.\ (\exists \pi \in \mathrm{Path}(\mathcal{S}) :\ \pi \not\models \varphi) \to \exists k :\ \pi|_k \not\models \varphi$
- Given an LTL formula φ, φ is a liveness formula iff $\forall \mathcal{S}.\ (\exists \pi \in \mathrm{Path}(\mathcal{S}) :\ \pi \not\models \varphi) \to |\pi| = \infty$
- All LTL formulas are either safety, liveness, or the AND of a safety and a liveness
  - being defined on paths, the counterexample is always a path
- Safety properties are those involving only **G**, **X**, true and atomic propositions
- Liveness are all those involving an **F**, or a **U** where the first formula is not the constant true
- Some formulas are both safety and liveness, like true, **G** true and so on

LTL Examples



$\mathcal{S} \models \mathbf{F}p$ since p holds in the first state

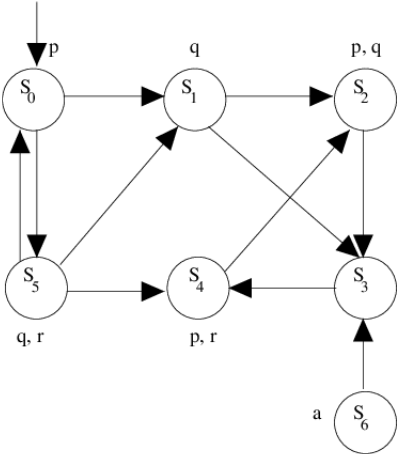
For full: let $\pi \in \mathrm{Path}(\mathcal{S})$

$\pi, 0 \models \mathbf{F}p$ with $j = 0$

recall: $\pi, i \models \mathbf{F}\Phi$ iff $\exists j \geq i.\ \pi, j \models \Phi$

$\pi, i \models p$ iff $p \in L(\pi(i))$

LTL Examples



$\mathcal{S} \not\models \mathbf{F}a$ since s_6 is not reachable from s_0

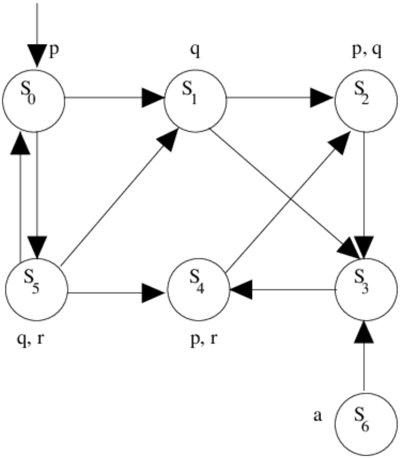
counterexample: $\pi = s_0 s_5 s0 s_5 \ldots$

For full: $\pi, 0 \not\models \mathbf{F}a$ as, for all $j \geq 0$, $a \notin L(\pi(j))$

Counterexample is infinite, thus this is a liveness property

Any finite prefix of π is not a counterexample

LTL Examples

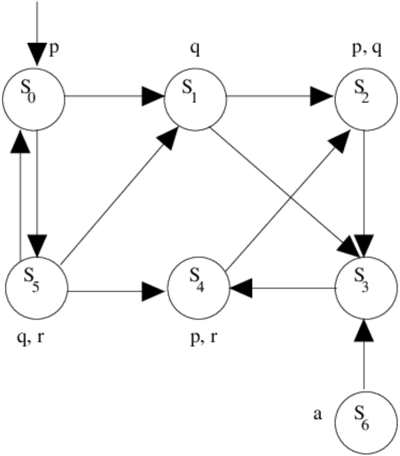


$\mathcal{S} \not\models \mathbf{G}p$ since there are many counterexamples, here is one:
$\pi = s_0 s_5 s_0 s_5 \ldots$
For full: $\pi, 0 \not\models \mathbf{G}p$ with $j = 1$

recall: $\pi, i \models \mathbf{G}\Phi$ iff $\forall j \geq i.\ \pi, j \models \Phi$
$\pi, i \models p$ iff $p \in L(\pi(i))$

Safety property, actually $\pi|_2$ is enough
Every path having $\pi|_2$ as a prefix is a counterexample

LTL Examples

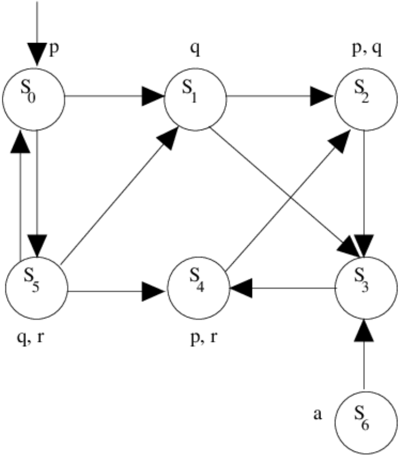


$\mathcal{S} \models \mathbf{G}\neg a$ since s_6 is not reachable from s_0

For full: let $\pi \in \mathrm{Path}(\mathcal{S})$ $\pi, 0 \models \mathbf{G}\neg a$ as the only state s with $a \in L(s)$ is s_6, which is not reachable from s_0

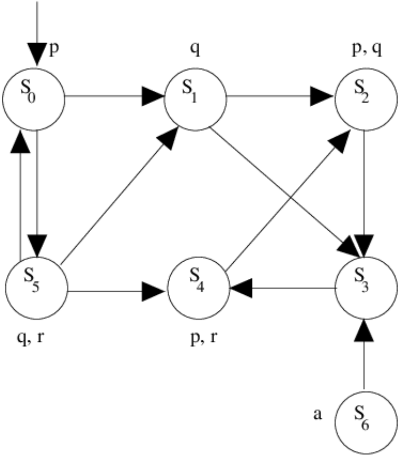
recall: $\pi \in \mathrm{Path}(\mathcal{S})$ implies $\pi(0) \in I$, thus $\pi(0) = s_0$ here

LTL Examples



$\mathcal{S} \models p\ \mathbf{U}\ q$ since $p \in L(s_0)$, $\mathrm{next}(s_0) = \{s_1, s_5\}$ and $q \in L(s_1) \wedge q \in L(s_5)$

LTL Examples

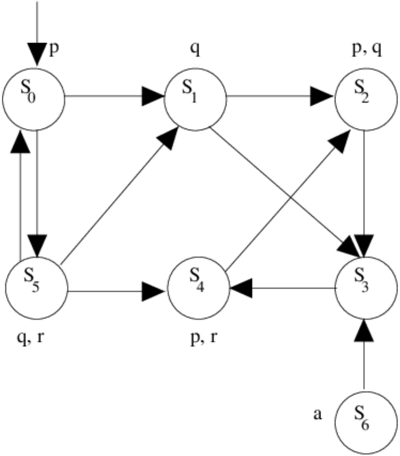


$\mathcal{S} \not\models p \textbf{ U } r$, a counterexample is $\pi = s_0 s_1 (s_2 s_3 s_4)$

Again this is a liveness formula, even if $\pi|_1$ would have been enough

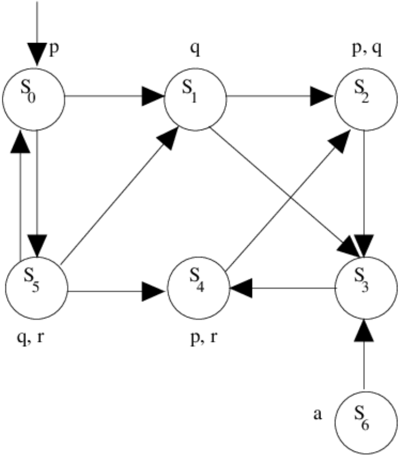
In fact, you have to consider all possible KSs...

LTL Examples



$\mathcal{S} \not\models \neg(p \ \mathbf{U} \ r)$, a counterexample is $\pi = (s_0 s_5)$
Thus it may happen that $\mathcal{S} \not\models \Phi$ and $\mathcal{S} \not\models \neg(\Phi)$
Instead, it is impossible that $\mathcal{S} \models \Phi$ and $\mathcal{S} \models \neg(\Phi)$

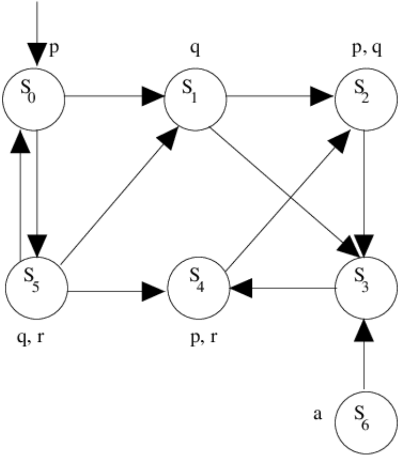
LTL Examples



$\mathcal{S} \not\models \mathbf{FG}p$, a counterexample is $\pi = s_0 s_1 (s_2 s_3 s_4)$

Again this is a liveness formula

LTL Examples

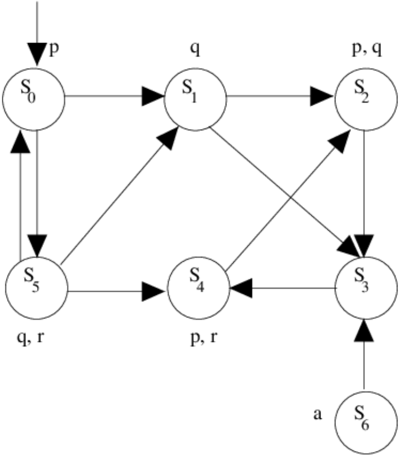


$\mathcal{S} \models \mathbf{GF}p$

All lassos are $s_0 s_5$ or $s_2 s_3 s_4$

In both such lassos, there are states in which p holds

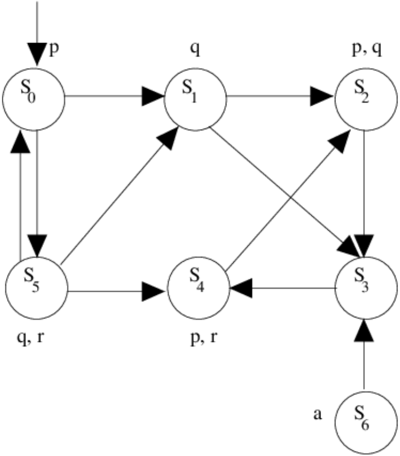
LTL Examples



$\mathcal{S} \models \mathbf{GF}p \vee \mathbf{FG}p$

Consequence of the two previous slides

LTL Examples



$\mathcal{S} \not\models \mathbf{G}(p\ \mathbf{U}\ q)$, a counterexample is $\pi = s_0 s_1 (s_2 s_3 s_4)$

$(p\ \mathbf{U}\ q)$ must hold at any reachable state

Ok in s_0, s_1, s_2, but not in s_3

LTL Non-Toy Examples

- Recall the Peterson's protocol: checking mutual exclusion is $\mathbf{G}(p \wedge q)$, being $p = \mathtt{P[1] = L3}, q = \mathtt{P[2] = L3}$
  - all invariants are of the form $\mathbf{G}P$, where P does not contain modal operators **X**, **U** or **F**
- Checking that both processes access to the critical section *infinitely often* is $\mathbf{GF}\ P[1] = L3 \wedge \mathbf{GF}\ P[2] = L3$
  - liveness property: no process is infinitely banned to access the critical section
- Even better: $\mathbf{G}\ (P[1] = L2 \rightarrow \mathbf{F}\ P[1] = L3)$
  - the same for the other process
  - since it is simmetric, this is actually enough

Equivalence Between LTL Properties

- Definition of equivalence between LTL properties:
  $\varphi_1 \equiv \varphi_2 \quad \text{iff} \quad \forall \mathcal{S}.\ \mathcal{S} \models \varphi_1 \Leftrightarrow \mathcal{S} \models \varphi_2$
  - equivalent: $\forall \sigma \ldots$
- Idempotency:
  - $\mathbf{FF}p \equiv \mathbf{F}p$
  - $\mathbf{GG}p \equiv \mathbf{G}p$
  - $p\ \mathbf{U}\ (p\ \mathbf{U}\ q) \equiv (p\ \mathbf{U}\ q)\ \mathbf{U}\ q \equiv p\ \mathbf{U}\ q$
- Absorption:
  - $\mathbf{GFG}p \equiv \mathbf{FG}p$
  - $\mathbf{FGF}p \equiv \mathbf{GF}p$
- Expansion (used by LTL Model Checking algorithms!):
  - $p\ \mathbf{U}\ q \equiv q \vee (p \wedge \mathbf{X}(p\ \mathbf{U}\ q))$
  - $\mathbf{F}p \equiv p \vee \mathbf{XF}p$
  - $\mathbf{G}p \equiv p \wedge \mathbf{XG}p$

CTL Syntax

$$\Phi ::= p \mid \Phi_1 \wedge \Phi_2 \mid \neg\Phi \mid (\Phi) \mid \mathbf{EX}\Phi \mid \mathbf{EG}\Phi \mid \mathbf{E}\Phi_1 \ \mathbf{U} \ \Phi_2$$

- Other derived operators (besides true, false, OR, etc):
  - $\mathbf{EF}\Phi = \mathbf{E}\mathrm{true} \ \mathbf{U} \ \Phi$
    - cannot be defined using $\mathbf{E}\neg\mathbf{G}\neg\Phi$, as this is not a CTL formula
    - actually, it is a CTL* formula (see later)
  - $\mathbf{AF}\Phi = \neg\mathbf{EG}\neg\Phi$, $\mathbf{AG}\Phi = \neg\mathbf{EF}\neg\Phi$, $\mathbf{AX}\Phi = \neg\mathbf{EX}\neg\Phi$
  - $\mathbf{A}\Phi_1 \ \mathbf{U} \ \Phi_2 = (\neg\mathbf{E}\neg\Phi_2 \ \mathbf{U} \ (\neg\Phi_1 \wedge \neg\Phi_1)) \wedge \neg\mathbf{EG}\neg\Phi_2$
  - $\Phi_1\mathbf{AU}\Phi_2 = \mathbf{A}\Phi_1\mathbf{U}\Phi_2$, $\Phi_1\mathbf{EU}\Phi_2 = \mathbf{E}\Phi_1\mathbf{U}\Phi_2$

Comparison with LTL Syntax

$$\Phi ::= \text{true} \mid p \mid \Phi_1 \wedge \Phi_2 \mid \neg\Phi \mid (\Phi) \mid \mathbf{X}\Phi \mid \Phi_1 \ \mathbf{U} \ \Phi_2$$

- Essentially, all temporal operators are preceded by either **E** or **G**
  - with some care for **U**

CTL Semantics

- Goal: formally defining when $\mathcal{S} \models \varphi$, being $\mathcal{S}$ a KS and φ a CTL formula
- This is true when, for all initial states $s \in I$ of $\mathcal{S}$, $s \pi \varphi$
  - thus, CTL is made of *state* formulas
  - LTL has *path* formulas
- To define when $s \models \varphi$, a recursive definition over the recursive syntax of CTL is provided
  - no need of an additional integer as for LTL syntax

CTL Semantics for $s, i \models \varphi$

- $\forall s \in S.\ s, i \models \mathrm{true}$
- $s \models p$ iff $p \in L(s)$
- $s \models \Phi_1 \wedge \Phi_2$ iff $s \models \Phi_1 \wedge s \models \Phi_2$
- $s \models \neg\Phi$ iff $s \not\models \Phi$
- $s \models \mathbf{EX}\Phi$ iff $\exists \pi \in \mathrm{Path}(\mathcal{S}, s).\ \pi(1) \models \Phi$
- $s \models \mathbf{EG}\Phi$ iff $\exists \pi \in \mathrm{Path}(\mathcal{S}, s).\ \forall j.\ \pi(j) \models \Phi$
- $s \models \mathbf{E}\Phi_1\ \mathbf{U}\ \Phi_2$ iff
  $\exists \pi \in \mathrm{Path}(\mathcal{S}, s) \exists k :\ \pi(k) \models \Phi_2 \wedge \forall j < k.\ \pi(j) \models \Phi_1$

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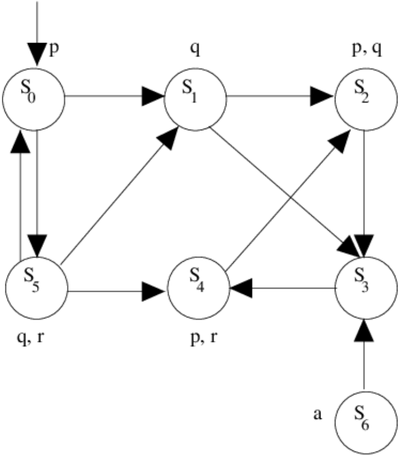
CTL Semantics for Added Operators

- It is easy to prove that:
  - $s \models \mathbf{AG}\Phi$ iff $\forall \pi \in \mathrm{Path}(\mathcal{S}, s).\ \forall j.\ \pi(j) \models \Phi$
  - $s \models \mathbf{AF}\Phi$ iff $\forall \pi \in \mathrm{Path}(\mathcal{S}, s).\ \exists j.\ \pi(j) \models \Phi$
  - analogously for **AU**, **AR**, **AW**
  - just replace $\forall$ with $\exists$ for **EF**, **ER**, **EW**
- As for CTL, for many formulas, it is silently required that paths are infinite
- So again transition relations in KSs must be total

Safety and Liveness Properties in CTL

- Some CTL formulas may be neither safety nor liveness
  - being defined on states, the counterexample may be an entire computation tree
- Safety properties are those involving only **AG**, **AX**, true and atomic propositions
- Some formulas are both safety and liveness, like true, **G** true and so on
- Liveness are formulas like **AF**, **AFAG**, **AU**
- **EF** or **EG** are neither liveness nor safety

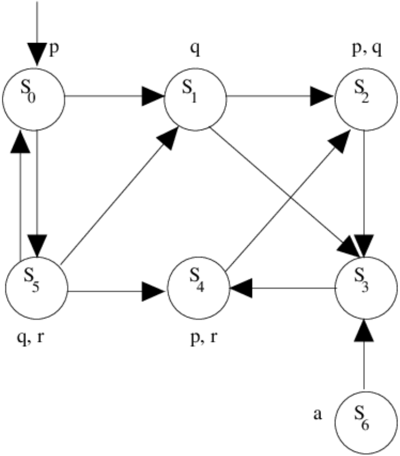
CTL Examples



$\mathcal{S} \models \mathbf{AF}p$ since p holds in the first state

For full: $s_0 \models \mathbf{F}p$ since $p \in L(s_0)$, thus, for all paths starting in s_0, p holds in the first state, so it holds eventually

CTL Examples



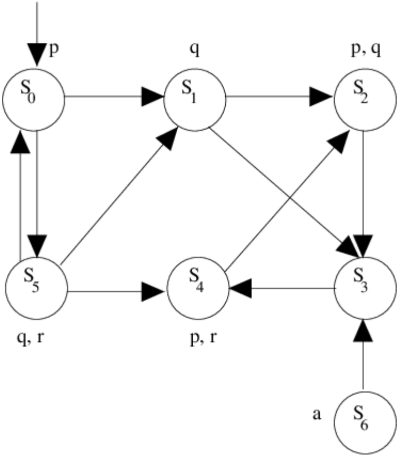
$\mathcal{S} \models \mathbf{EF}p$ for the same reason as above

If it holds for all paths, then it holds for one path

$\mathbf{AF}\Phi \rightarrow \mathbf{EF}\Phi$

The same holds for the other temporal operators $\mathbf{G}, \mathbf{U}$ etc

CTL Examples



$\mathcal{S} \not\models$ **EF**a since s_6 is not reachable

Note that the counterexample cannot be a single path

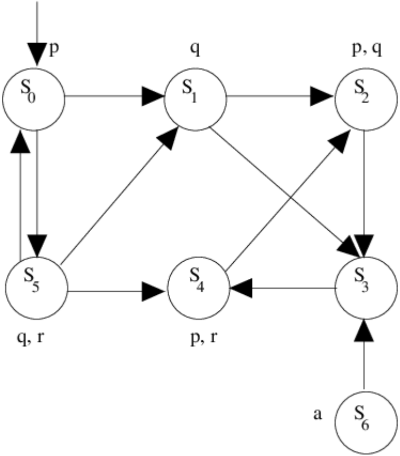
Since it would not enough to disprove existence

The full reachable graph must be provided

One could also show the tree of all paths

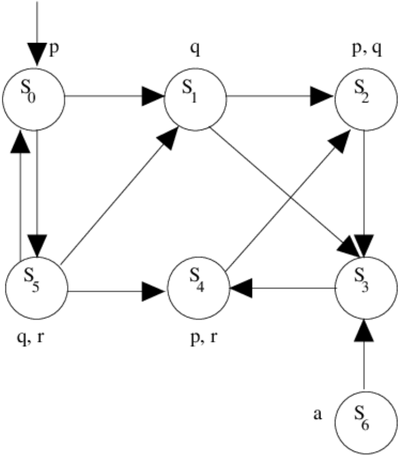
Neither safety nor liveness

CTL Examples



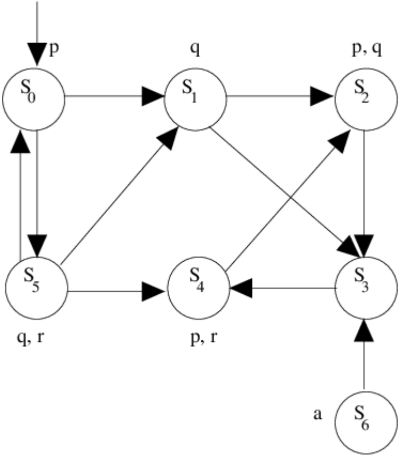
$\mathcal{S} \models \mathbf{A}(p\ \mathbf{U}\ q)$ since $p \in L(s_0)$, $\mathrm{next}(s_0) = \{s_1, s_5\}$ and $q \in L(s_1) \wedge q \in L(s_5)$

CTL Examples



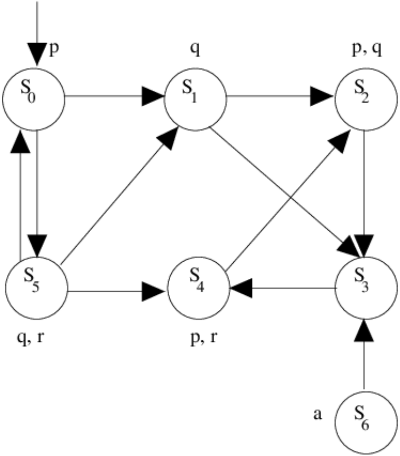
$\mathcal{S} \not\models \mathbf{A}(p \; \mathbf{U} \; r)$, a counterexample is $\pi = s_0 s_1 (s_2 s_3 s_4)$

CTL Examples



$\mathcal{S} \models \mathbf{E}(p \ \mathbf{U} \ r)$, an example is $\pi = (s_0 s_5)$

CTL Examples

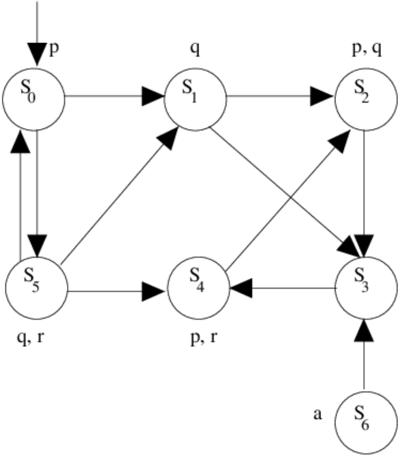


$\mathcal{S} \not\models \neg\mathbf{E}(p\ \mathbf{U}\ r)$, a counterexample is $\pi = (s_0 s_5)$

In fact, $\mathcal{S} \not\models \Phi$ iff $\mathcal{S} \models \neg(\Phi)$

No hidden quantifier...

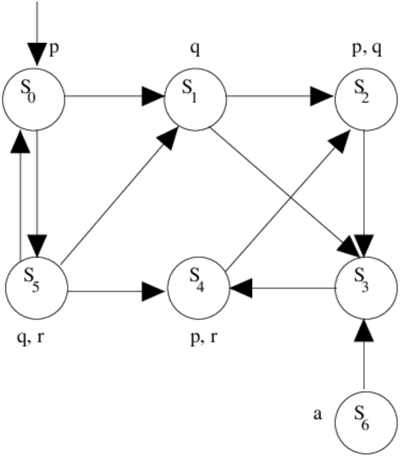
CTL Examples



$\mathcal{S} \not\models$ **AFAG**p, a counterexample is $\pi = s_0 s_1 (s_2 s_3 s_4)$

This is a liveness formula

CTL Examples

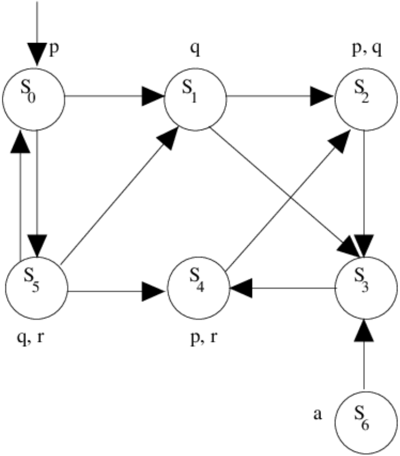


$\mathcal{S} \not\models$ **EFEG**p, a counterexample is again a computation tree

All lassos are $s_0 s_5$ or $s_2 s_3 s_4$

In both such lassos, there are states in which p does not hold

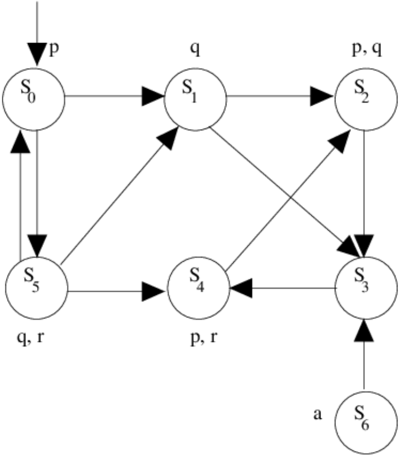
CTL Examples



$\mathcal{S} \not\models$ **AFEG**p, a counterexample is again a computation tree

Since $\mathcal{S} \not\models$ **EFEG**p...

CTL Examples



$\mathcal{S} \not\models$ **EFAG**p, a counterexample is again a computation tree

Since $\mathcal{S} \not\models$ **EFEG**p...

CTL Non-Toy Examples

- Recall the Peterson's protocol: checking mutual exclusion is $\mathbf{AG}(p \wedge q)$, being $p = \mathtt{P[1] = L3}, q = \mathtt{P[2] = L3}$
  - equivalent to LTL $\mathbf{G}p$
- It is always possible to restart: $\mathbf{AGEF}\ P[1] = L0 \wedge \mathbf{AGEF}\ P[2] = L0$

CTL vs. LTL: a Comparison

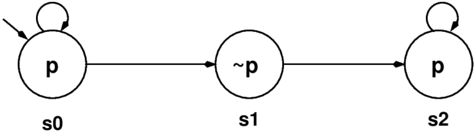
- Recall that $\varphi_1 \equiv \varphi_2$ iff $\forall \mathcal{S}. \, \mathcal{S} \models \varphi_1 \Leftrightarrow \mathcal{S} \models \varphi_2$
  - also holds (w.l.g.) when φ_1 is LTL and φ_2 is CTL
- Of course, some CTL formulas cannot be expressed in LTL
  - it is enough to put an **E**, since LTL always universally quantifies paths
  - so, there is not an LTL φ s.t. $\varphi \equiv \mathbf{EG}p$
    - no, $\mathbf{F}\neg p$ is not the same, why?
- So, one might think: LTL is contained in CTL
  - simply replace each temporal operator **O** with **AO**, that's it
  - let $\mathcal{T}$ be a translator doing this
  - for any LTL formula φ, $\varphi \equiv \mathcal{T}(\varphi)$
  - actually, $\mathbf{G}p \equiv \mathcal{T}(\mathbf{G}p) = \mathbf{AG}p$

CTL vs. LTL: a Comparison

- Theorem. Let φ be an LTL formula. Then, either i) $\varphi \equiv \mathcal{T}(\varphi)$ or ii) there does not exist a CTL formula ψ s.t. $\varphi \equiv \psi$
  - idea of proof: replacing with **E** is of course not correct, and temporal operators on paths are the same
- Corollary. There exists an LTL formula φ s.t., for all CTL formulas ψ, $\varphi \not\equiv \psi$
- Proof of corollary:
  - by the theorem above and the definitions, we need to find
    1. an LTL formula φ
    2. a KS $\mathcal{S}$
  - where $\mathcal{S} \models \varphi$ and $\mathcal{S} \not\models \mathcal{T}(\varphi)$
    - viceversa is not possible

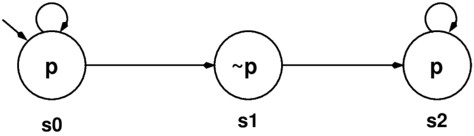
CTL vs. LTL: a Comparison

- For example, as for the LTL formula, we may take $\varphi = \mathbf{FG}p$
  - note instead that $\mathbf{GF}p \equiv \mathbf{AGAF}p$
- For example, as for the KS $\mathcal{S}$, we may take



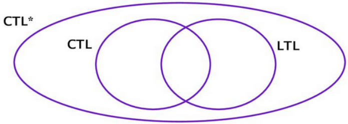
- We have that $\mathcal{S} \models \mathbf{FG}p$, but $\mathcal{S} \not\models \mathbf{AFAG}p$
- Thus, CTL requires "more" than the corresponding LTL

CTL vs. LTL: a Comparison



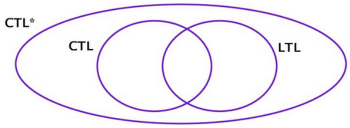
- $\mathcal{S} \not\models$ **AFAG**p means that
  $\neg(\forall \pi \in \mathrm{Path}(\mathcal{S}).\ \exists j:\ \forall \rho \in \mathrm{Path}(\mathcal{S}, \pi(j)).\ \forall k.\ p \in \rho(k))$
  $= \exists \pi \in \mathrm{Path}(\mathcal{S}).\ \forall j:\ \exists \rho \in \mathrm{Path}(\mathcal{S}, \pi(j)).\ \exists k.\ p \notin \rho(k)$
  - the path π is a loop on s_0...
- $\mathcal{S} \models$ **FG**p means that $\forall \pi \in \mathrm{Path}(\mathcal{S}).\ \exists j:\ \forall k \geq j.\ p \in \pi(k)$
- Thus, there is not a CTL formula equivalent to **FG**p
- Furthermore, there is not an LTL formula equivalent to **AFAG**p

CTL, LTL and CTL*



- CTL* introduced in 1986 (Emerson, Halpern) to include both CTL and LTL
- No restrictions on path quantifiers to be 1-1 with temporal operators, as in CTL
- State formulas: $\Phi ::= \mathrm{true} \mid p \mid \Phi_1 \wedge \Phi_2 \mid \neg\Phi \mid \mathbf{A}\Psi \mid \mathbf{E}\Psi$
- Path formulas: $\Psi ::= \Phi \mid \Psi_1 \wedge \Psi_2 \mid \neg\Psi \mid \Psi_1 \mathbf{U} \Psi_2 \mid \mathbf{F}\Psi \mid \mathbf{G}\Psi$

CTL, LTL and CTL*



- The intersection between CTL and LTL is both syntactic and "semantic"
- Some formulas are both CTL and LTL in syntax: all those involving only boolean combinations of atomic propositions
- "Semantic" intersection: some LTL formulas may be expressed in CTL and vice versa, using different syntax
  - **AGAF**p and **GF**p
  - **AG**p and **G**p
  - etc

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Peterson Protocol in Promela

```
bool turn, flag[2];
byte ncrit;

active [2] proctype user()
{
 assert(_pid == 0 || _pid == 1);
again:
 flag[_pid] = 1;
 turn = _pid;
 (flag[1 - _pid] == 0 || turn == 1 - _pid);
 ncrit++;
 assert(ncrit == 1); /* critical section */
 ncrit--;
 flag[_pid] = 0;
 goto again
}
```

Dijkstra Protocol in Promela

```
#define p   0
#define v   1
chan sema = [0] of { bit }; /* rendez-vous */

proctype dijkstra()
{    byte count = 1; /* local variable */
     do
     :: (count == 1) -> sema!p; count = 0
     /* send 0 and blocks, unless some other
        proc is already blocked in reception */
     :: (count == 0) -> sema?v; count = 1
     /* receive 1, same as above */
     od
}
```

Dijkstra Protocol in Promela

```
proctype user()
{   do
    :: sema?p;
       /*      critical section */
       sema!v;
       /* non-critical section */
    od
}

init
{   run dijkstra();
    run user(); run user(); run user()
}
```

SPIN Simulation

Almost equal to Murphi one

```
void Make_a_run(NFSS 𝒩)
{
 let 𝒩 = ⟨S, {s₀}, Post⟩;
 s_curr = s₀;
 if (some assertion fail in s_curr))
  return with error message;
 while (1) { /* loop forever */
  if (Post(s_curr) = ∅)
   return with deadlock message;
  s_next = pick_a_state(Post(s_curr));
  if (some assertion fail in s_curr))
   return with error message;
  s_curr = s_next;
 }
}
```

SPIN Verification

- Able to answer to the following questions:
  - is there a deadlock (invalid end state)?
  - are there reachable assertions which fail (safety)?
  - is a given LTL formula (safety or liveness) ok in the current system?
  - is a given neverclaim (safety or liveness) ok in the current system?
- It is possible to specify some side behaviours:
  - is sending to a full channel blocking, or the message is dropped without blocking?
- It may report unreachable code
  - Promela statements in the model which are never executed

SPIN Verification

- Similar to Murphi:
  1. the SPIN compiler (`SrcXXX/spin -a`) is invoked on `model.prm` and outputs 5 files:
     - `pan.c`, `pan.h`, `pan.m`, `pan.b`, `pan.t` (unless there are errors...)
  2. the 5 files given above are compiled with a C compiler
     - it is sufficient to compile `pan.c`, which includes all other files
     - in this way, an executable file `model` is obtained
  3. just execute `model`
     - option `--help` gives an overview of all possible options

Standard Recursive DFS

```
HashTable Visited = ∅;

DFS(graph G = (V, E), node v)
{
  Visited := Visited ∪ v;
  foreach v' ∈ V t.c. (v, v') ∈ E {
    if (v' ∉ Visited)
      DFS(G, v');
  }
}
```

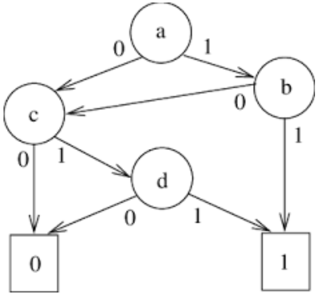
Iterative DFS

```
DFS(graph G = (V, E))
{
  s := init; i := 1; depth := 0;
  push(s, 1);
Down:
  if (s ∈ Visited)
    goto Up;
  Visited := Visited ∪ s;
  let S' = {s' | (s, s') ∈ E};
  if (|S'| >= i) {
    s := i-th element in S';
    increment i on the top of the stack;
    push(s, 1);
    depth := depth + 1;
    goto Down;
  }
```

Iterative DFS

```
Up:
  (s, i) := pop();
  depth := depth - 1;
  if (depth > 0)
    goto Down;
}
```

Binary Decision Diagrams



Represented function: $f(a, b, c, d) = ab + \bar{a}cd + a\bar{b}cd$

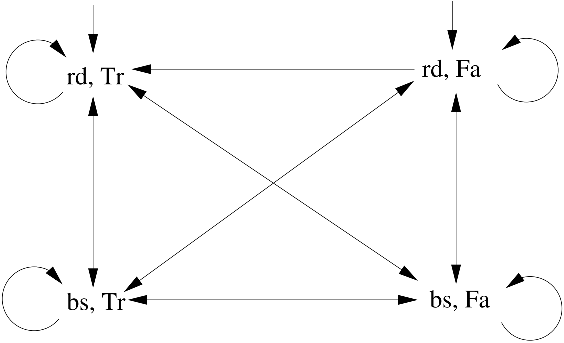
- recall that $+$ is OR, $\cdot$ is AND, $\bar{\cdot}$ is negation

NuSMV Input Language

Taken from `examples/smv-dist/short.smv`

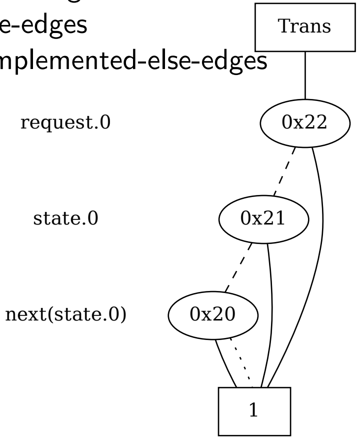
```
MODULE main
VAR
  request : {Tr, Fa}; -- same as saying boolean
                      -- (stand for True and False)
  state : {ready, busy};
ASSIGN
  init(state) := ready;
  next(state) := case
                   state = ready & (request = Tr): busy;
                   1 : {ready,busy};
                 esac;
SPEC
  AG((request = Tr) -> AF state = busy)
```

Automata for `short.smv`: I and R

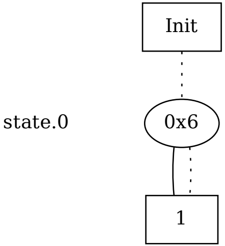


OBDDs for `short.smv`: R

Straight lines are then-edges
Dashed lines are else-edges
Dotted lines are complemented-else-edges



OBDDs for `short.smv`: I



NuSMV Input Language

```
MODULE user(semaphore)
VAR
  state : {idle, entering, critical, exiting};
ASSIGN
  init(state) := idle;
  next(state) :=
    case
      state = idle: entering;
      state = entering & !semaphore: critical;
      state = critical: {critical, exiting};
      state = exiting: idle;
      TRUE : state;
    esac;
```

```
next(semaphore) :=
  case
   state = entering: TRUE;
   state = exiting: FALSE;
   TRUE: semaphore;
  esac;
```

NuSMV Input Language

```
MODULE main
VAR
  semaphore : boolean;
  proc1 : process user(semaphore);
  proc2 : process user(semaphore);
ASSIGN
  init(semaphore) := FALSE;

SPEC
  AG(!(proc1.state = critical & proc2.state = critical))

LTLSPEC
  G F proc1.state = critical
```

Computation of Least (Minimum) Fixpoint

```
OBDD lfp(MuFormula T) /* μZ.T(Z) */
{
  Q = λx.0;
  Q' = T(Q);
  /* T clearly says where Q must be replaced */
  /* e.g.: if μZ.λx.f(x) ∨ Z(x), then
     Q' = λx.f(x) ∧ Q(x) */
  while (Q ≠ Q') {
    Q = Q';
    Q' = T(Q);
  }
  return Q; /* or Q', they are the same... */
}
```

Computation of Greatest (Maximum) Fixpoint

```
OBDD gfp(NuFormula T) /* νZ.T(Z) */
{
  Q = λx.1;
  Q' = T(Q);
  while (Q ≠ Q') {
    Q = Q';
    Q' = T(Q);
  }
  return Q;
}
```

CTL Model Checking

```
bool checkCTL(KS 𝒮, CTL φ) {
   let 𝒮 = ⟨S, I, R, L⟩;
   B = LblSt(φ);
   return λx. I(x) ∧ ¬B(x) = λx. 0;
}
OBDD LblSt(CTL φ) { /* also 𝒮 = ⟨S, I, R, L⟩ */
  if (∃p ∈ AP. φ = p) return λx. p(x);
  else if (φ = ¬ϕ) return λx. ¬LblSt(ϕ)(x);
  else if (φ = ϕ1 ∧ ϕ2)
   return λx.LblSt(ϕ1)(x)∧LblSt(ϕ2)(x);
  else if (φ = EXϕ)
   return λx. ∃y : R(x, y)∧LblSt(ϕ)(y);
  else if (φ = EGϕ)
   return gfp(νZ. λx. LblSt(ϕ)(x) ∧ (∃y : R(x, y) ∧ Z(y)));
  else if (φ = ϕ1 EU ϕ2)
   return lfp(μZ. λx. LblSt(ϕ2)(x)∨
     (LblSt(ϕ1)(x) ∧ (∃y : R(x, y) ∧ Z(y))));
}
```

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