

Automated Verification of Cyber-Physical Systems

A.A. 2023/2024

Corso di Laurea Magistrale in Informatica

Basic Notions

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General Info for This Class

- Automated Verification of Cyber-Physical Systems is an elective course for the Master Degree in Computer Science
- Lecturer: Igor Melatti
- Where to find these slides and more:
 - https://igormelatti.github.io/aut_ver_cps/20232024/index_eng.html
 - also on MS Teams: "DT0759: Automated Verification of Cyber-Physical Systems (2023/24)", code **ramh3r4**
- 2 classes every week, 2 hours per class



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Rules for Exams

- The exam consists in either reviewing a research paper or working on a project
- Each student may choose one between the two options
- Project: perform verification of a given cyber-physical system
 - also in small teams (max 3 students)
 - each team may choose one among the ones selected by lecturer
 - or may propose one (but wait for lecturer approval!)
 - each team will have to discuss its project with slides
- Paper: read a conference or journal paper and present it with slides
 - each student may choose one among the ones selected by lecturer
 - or may propose one (but wait for lecturer approval!)



Model Checking Problem

- Input: a system $\mathcal{S}$ and (at least) a property φ
 - more precisely, a *model* of $\mathcal{S}$ must be provided
 - that is, $\mathcal{S}$ must be described in some suitable language
- Output:
 - PASS** $\mathcal{S}$ satisfies φ , i.e., $\mathcal{S} \models \varphi$
 - the system $\mathcal{S}$ is correct w.r.t. the property φ
 - mathematical certification, much better than, e.g., testing
 - FAIL** $\mathcal{S}$ does not satisfy φ , i.e., $\mathcal{S} \not\models \varphi$
 - the system $\mathcal{S}$ is buggy w.r.t. the property φ
 - a *counterexample* providing evidence of the error is also returned



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Model Checking vs. Other Verification Techniques

- Model checking is fully automatic
 - a model checker only needs the description of $\mathcal{S}$ and the property φ
 - “press button and go”
 - this is not true for other verification tools such as proof checkers, which require human intervention in the process
- Model checking is correct for both PASS and FAIL
 - unless the description of $\mathcal{S}$, or the property φ , are wrong
 - this is not true for other verification techniques such as testing, which only guarantees the FAIL result
 - a buggy system may pass all tests, because the error is in some *corner case*



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Model Checking Shortcomings

- Only works for finite-state systems
 - typical example: you may verify a system with 3, 4 or 5 processes, but not with n processes, for a generic n
- Requires skilled personnel to write descriptions (and properties)
 - must know both the model checker language and the system
 - however, less skilled than a proof checker user
 - very few exceptions in which the model is automatically extracted from the system
 - also direct translations from digital circuits to NuSMV are available
- Very resource demanding
 - besides PASS and FAIL, also OutOfMem and OutOfTime are expected results...
 - bounded model checking: PASS is limited to execution up to a given number of steps



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Model Checking Algorithms

Two main categories:

Explicit visit the graph induced by the description of $\mathcal{S}$

- very good for invariants and LTL model checking of communication protocols
- on-the-fly generation of the graph: only the reachable states are stored, the adjacency matrix is implicitly given by the description of $\mathcal{S}$
- Murphi, SPIN

Symbolic represent sets of states and transition relations as OBDDs

- very good for LTL and CTL model checking of hardware-like systems
- all translated into a boolean formula
- also SAT tools may be used (bounded model checking)



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Cyber-Physical Systems

- A Cyber-Physical System (CPS) is a system where a physical system is controlled and/or monitored by a software
- They are either partially or fully autonomous
 - we will mainly deal with fully autonomous CPSs
- Examples are everywhere:
 - Internet of Things devices
 - Unmanned Autonomous Vehicles
 - Drones
 - Medical Devices
 - Embedded Systems
 - ...

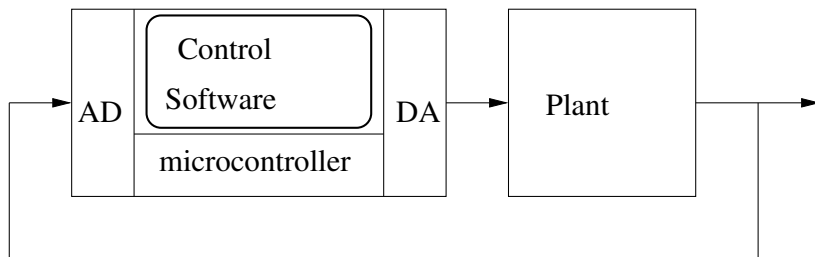


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Cyber-Physical Systems with Controllers



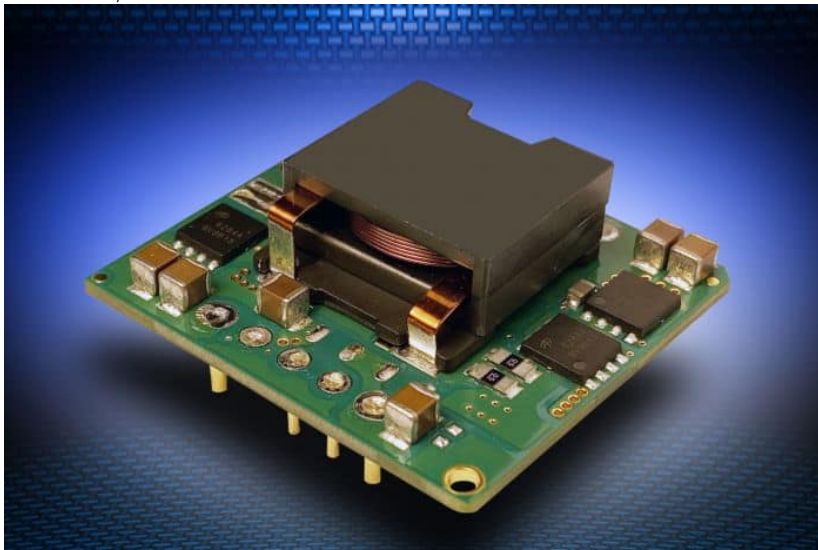
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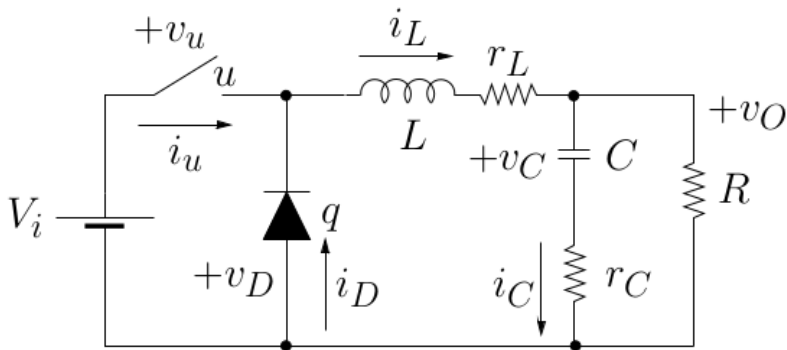
CPSs with Controllers: Classical Examples

Buck DC/DC Converter



CPSs with Controllers: Classical Examples

Buck DC/DC Converter



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CPSs with Controllers: Classical Examples

Continuous time dynamics

$$\dot{i}_L = a_{1,1}i_L + a_{1,2}v_O + a_{1,3}v_D \quad (1)$$

$$\dot{v}_O = a_{2,1}i_L + a_{2,2}v_O + a_{2,3}v_D \quad (2)$$

$$q \rightarrow v_D = R_{\text{on}}i_D \quad (3) \qquad \bar{q} \rightarrow v_D = R_{\text{off}}i_D \quad (7)$$

$$q \rightarrow i_D \geq 0 \quad (4) \qquad \bar{q} \rightarrow v_D \leq 0 \quad (8)$$

$$u \rightarrow v_u = R_{\text{on}}i_u \quad (5) \qquad \bar{u} \rightarrow v_u = R_{\text{off}}i_u \quad (9)$$

$$v_D = v_u - V_{in} \quad (6) \qquad i_D = i_L - i_u \quad (10)$$

where:

- i_L, v_O are state variables
- $u \in \{0, 1\}$ is the action



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CPSs with Controllers: Classical Examples

Discrete time dynamics with sampling time T

$$i_L' = (1 + Ta_{1,1})i_L + Ta_{1,2}v_O + Ta_{1,3}v_D \quad (11)$$

$$v_O' = Ta_{2,1}i_L + (1 + Ta_{2,2})v_O + Ta_{2,3}v_D. \quad (12)$$

$$q \rightarrow v_D = R_{\text{on}}i_D \quad (13)$$

$$q \rightarrow i_D \geq 0 \quad (14)$$

$$u \rightarrow v_u = R_{\text{on}}i_u \quad (15)$$

$$v_D = v_u - V_{in} \quad (16)$$

$$\bar{q} \rightarrow v_D = R_{\text{off}}i_D \quad (17)$$

$$\bar{q} \rightarrow v_D \leq 0 \quad (18)$$

$$\bar{u} \rightarrow v_u = R_{\text{off}}i_u \quad (19)$$

$$i_D = i_L - i_u \quad (20)$$



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CPSs with Controllers: Classical Examples

- Goal: keep v_O in a desired safe interval
 - typically, $5 - 0.01V \leq v_O \leq 5 + 0.01V$
- Notwithstanding the input voltage V_i and the resistance R may vary in some given interval
 - typically, $R = 5 \pm 25\% \Omega$, $V_i = 15 \pm 25\% V$
- Effectively used in laptops: from battery voltage (V_i) to laptop processor voltage (v_O)



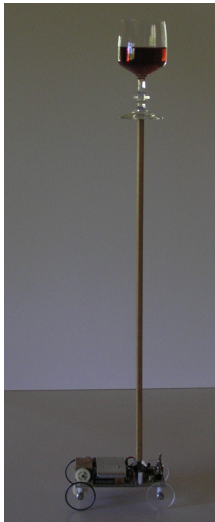
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CPSs with Controllers: Classical Examples

Inverted Pendulum



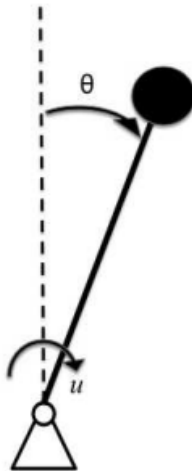
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CPSs with Controllers: Classical Examples

Inverted Pendulum



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CPSs with Controllers: Classical Examples

Continuous time dynamics

$$\ddot{\theta} = \frac{g}{l} \sin \theta + \frac{1}{ml^2} Fu$$

where:

- θ is the state variable
- $u \in \{0, 1\}$ is the action
- m, l, F are system parameters



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CPSs with Controllers: Classical Examples

Continuous time dynamics

$$\dot{x}_1 = x_2 \quad (21)$$

$$\dot{x}_2 = \frac{g}{l} \sin x_1 + \frac{1}{ml^2} Fu \quad (22)$$

Discrete time dynamics with sampling time T

$$x_1' = x_1 + Tx_2 \quad (23)$$

$$x_2' = x_2 + T\frac{g}{l} \sin x_1 + T\frac{1}{ml^2} Fu \quad (24)$$



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In This Course

To deal with cyber-physical systems:

- Probabilistic Model Checking
 - rather than “are there errors?”, it is “is the error probability low enough?”
 - which entails “what is the error probability?”
 - the system is probabilistic, i.e., a Markov Chain
- Statistical Model Checking
 - rather than “are there errors?”, it is “is the error probability low enough?”
 - which entails “what is the error probability?”
 - the system may be a non-probabilistic simulator
 - the answer is given with some statistical confidence
 - bridge between testing and verification



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In This Course

To deal with cyber-physical systems:

- System Level Formal Verification
 - directly use a simulator instead of describing the system within the model checker
 - this will also need some background on systems simulation
 - bridge between testing and verification
- Automatic Synthesis of Controllers
 - rather than “are there errors in this system?”, it is “generate a controller so that errors are avoided”



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Systems Verification

Summing up:

- ① start from requirements
- ② develop some (partial or final) solution
 - you may “complicate” such steps at wish
- ③ *verify* that the current solution fulfills the starting requirements
 - you may need to change the requirements (they could be wrong too, or they may have been changed)
 - recall that verification may also be done during the intermediate developing steps



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How Verification is Performed

- Method number 1: *Testing*
 - ① you have the actual system (or a part of it)
 - ② you feed it with predetermined *inputs*
 - ③ you check if *outputs* are the expected ones
 - “expected” w.r.t. the requirements
 - ④ if there is one output different from the expected one, then we have an error
 - ⑤ you correct it and start over again
- Method number 1 bis: *Simulation*
 - instead of using the actual system, you have a (*software*) *simulator*
 - especiall useful for hardware or for physical parts
 - if you want to do testing on hardware, you need to actually build it, which may be expensive



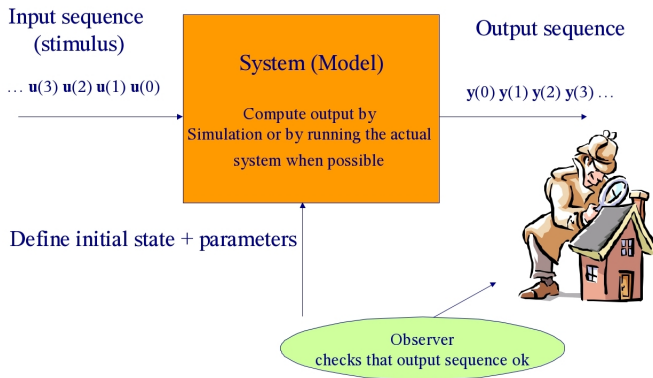
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How Verification is Performed

An approximate answer BUG HUNTING: Testing + Simulation



How Verification is Performed

- Both testing and simulation may be performed in refined ways
- In fact, the *testing plan* (the predetermined sequence of inputs) may be computed using dedicated algorithms so that *coverage* is maximized
 - we will get back soon on this concept
- This is the most challenging and important step for such techniques



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Testing and Simulation: Pro and Cons

Pro

- (Relatively) easy to implement
 - easier than the other methods we will consider here
- Largely used in industry
 - in most cases, testing and/or simulation are the *only* verification methods used

Cons

- They can prove that a system *has* errors, but cannot prove that a system *does not have* errors
- Cannot be used to prove generic formal properties
- The coverage of the “input space” is low
- Errors are frequently detected when it is too



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Testing and Simulation: Cons

They can prove that a system *has* errors, but cannot prove that a system *does not have* errors

- If an error is detected, then the system must be corrected, happy to have discovered it
- Otherwise, *we cannot conclude anything*
- That is, **we cannot say that the system is error-free**
- In fact, having not be able to spot errors does not imply that there are no errors



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Testing and Simulation: Cons

Cannot be used to prove generic formal properties

- This is a consequence of the previous slide
- As an example: in an operating system, is it true that mutual exclusion is enforced for 2 given processes?
- In order to test such a property you would have to modify the system itself
 - so that the output contains something like “propriety violated” or “property ok”
- But even in this case, we cannot draw a formal statement on the validity of the property
- Again, not finding a violation does not imply there are no violations



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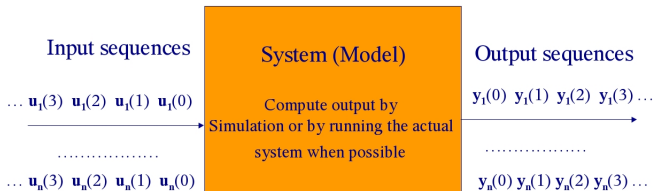


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Testing and Simulation: Cons

The coverage of the “input space” is low

- A successful testing phase should consider “all what may happen” to the system in a real-world environment
- This would need too much tests or simulations



- The n in the figure may easily be 10^6 and more; outputs must also be checked



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Testing and Simulation: Cons

The coverage of the “input space” is low

- This also has another bad consequence
- Testing and simulation find the “easy” errors
 - the most frequent ones
 - i.e., those that are caused by many (different) input sequences
- Instead, *corner cases* usually go undetected
 - i.e., errors that are caused by a few (or even single) input sequences are usually not found



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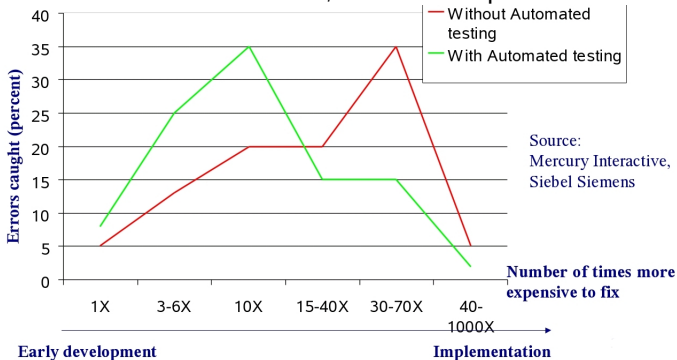


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Testing and Simulation: Cons

Errors are frequently detected when it is too late

- This is a consequence of the previous point: you need many tests to get a reasonable coverage and discover possible corner cases
- The later an error is found, the more expensive the correction



Formal Verification

- To solve the above underlined problems, we should consider *all* inputs
- That is, all possible system *evolutions*
 - of course, testing and simulation only consider *some* evolutions: those “activated” by inputs chosen by the testing plan in use
- A possible way to do this is to prove a dedicated theorem, stating that the system is correct for all inputs
- For sorting, this could be done (and it is actually done in Algorithms textbooks...)
- For other cases (e.g., microprocessor design), it would be too difficult or time consuming
- Thus, techniques of *formal verification* have been developed



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Formal Verification Methods

- A set of (heterogeneous) techniques which make possible the impossible
- That is, algorithms able to generate and analyze *all* system evolutions
 - so, they provide a *mathematical certification* of correctness (not achievable with testing/simulation)
 - also for generic properties, like mutual exclusion
- Actually, the problem of verifying a given system w.r.t. a given property is *undecidable*
 - the property to be verified may be: is this system always terminating?
- So, there will be some (acceptable in many cases) limitations



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Is Formal Verification Useful?

- There are many techniques available for formal verification
- Applying any of these techniques is usually much more difficult than testing/simulation
 - both in terms of personnel and notions required
- So, why to do this?
- Because there are many cases in which testing/simulation simply *are not enough*
 - for both economic and safety reasons



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Is Formal Verification Useful?

- **Safety-critical** systems: failures may affect humans
 - public transport software controllers (if an automatic pilot of an airplane has a failure...)
 - trains crossing
 - ABS for cars
 - ...
- For most of such systems, formal verification is **mandatory** by law
 - ESA (European Space Agency)
 - IEC (International Electrotechnical Commission)



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Is Formal Verification Useful?

- **Mission-critical** systems: failures cause huge economic losses
 - automatic space probes
 - logistics
 - communication networks
 - microprocessors
 - ...
- Internal company regulations often make formal verification **mandatory** as well



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Is Formal Verification Useful?

- Also for systems which are neither safety nor mission critical: there are economic motivations to use formal verification
- Using testing/simulations, errors are eventually discovered
- The problem is that they may be found *late*
 - this is a consequence of the low coverage issue
- So late, that often errors are found *after* the system has been deployed, i.e., when it is already used by its final users
 - for, e.g., a *word processor*, it is annoying, but we are somewhat used to software updates to fix bugs
 - this is not always possible or easy
 - e.g., a legacy software out of support



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Is Formal Verification Useful?

- Hardware circuits: to “write” a circuit on silicon is the most expensive part of the developing process
- So, finding an error after having written the circuit entails a huge economic loss
- This also holds for other systems, when the developing process is lengthy
- In fact, finding a late error may cause going again through preceding developing phases
 - less competitiveness on the market
 - for both being late and for augmented costs



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Formal Verification Methodologies: a Classification

There are two macro-categories:

- *Interactive methods*
 - as the name suggests, not (fully) automatic
 - human intervention is typically required
 - in this course, we do not deal with such techniques
- *Automatic methods*
 - only human intervention is to *model* the system
- There also exist hybridations among the two categories



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Interactive Methods

- Also called *proof checkers*, *proof assistants* or *high-order theorem provers*
- Tools which helps in building a mathematical proof of correctness for the given system and property
- **Pros**
 - virtually no limitation to the type of system and property to be verified
- **Cons**
 - highly skilled personnel is needed
 - both in mathematical logic and in deductive reasoning
 - needed to “help” tools in building the proof



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Interactive Methods

- Used for projects with high budgets
- For which the automatic methods limitations are not acceptable
 - used, e.g., to prove correctness of microprocessor circuits or OS microkernels
- Some tools in this category (see https://en.wikipedia.org/wiki/Proof_assistant):
 - HOL
 - PVS
 - Coq



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Automatic Methods

- Commonly dubbed *Model Checking*
- Model Checking software tools are called *model checkers*
- There are some tens model checkers developed; the most important ones are listed in https://en.wikipedia.org/wiki/List_of_model_checking_tools
- Many are freely downloadable and modifiable for research and study purposes
- Research area with many achievements in over 30 years

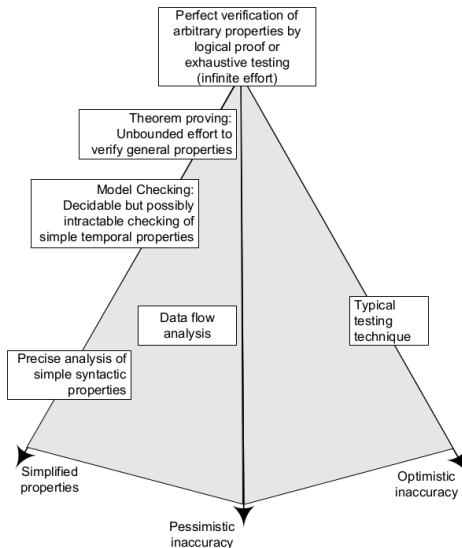


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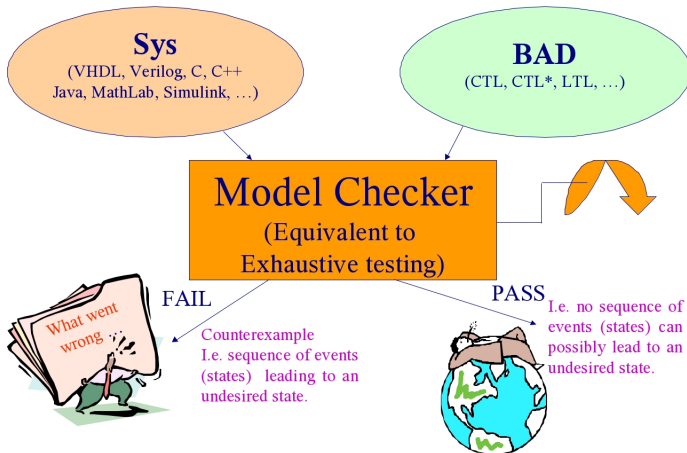


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Verification Tradeoffs



The Model Checking Dream

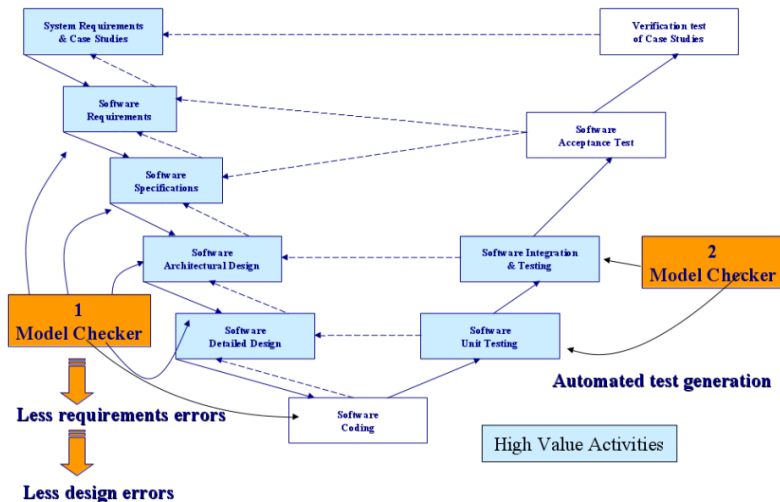


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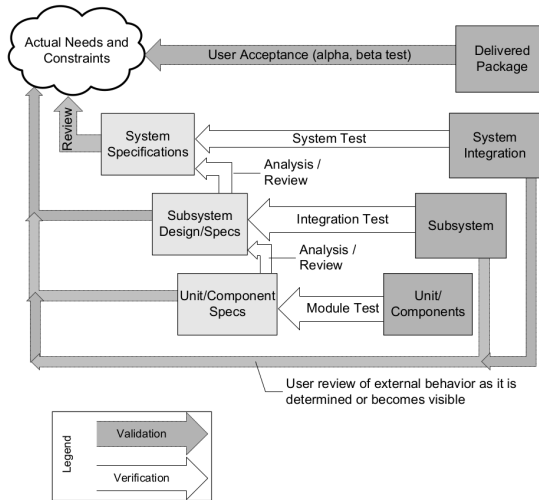


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The Model Checking Dream



Also Keep This in Mind



Actual Model Checking

- In order to have this computationally feasible, we need a strong assumption on the system under verification (SUV)
- I.e., it must have a *finite number of states*
 - *Finite State System* (FSS)
- In this way, model checkers “simply” have to implement reachability-related algorithms on graphs
- Such finite state assumption, though strong, is applicable to many interesting systems
 - that is: many systems are actually FSSs
 - or they may be approximated as such
 - or a part of them may be approximated as such



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What Is a *State*?

- There are many notions of “state” in computer science
- Model checking states are *not* the ones in UML-like state diagrams
- Model checking states are similar to operational semantics states
- That is: suppose that a system is “described” by n variables
- Then, a state is an assignment to all n variables
 - given $D_1, \dots, D_n$ as our n variables domains, then a state is $s \in \times_{i=1}^n D_i$



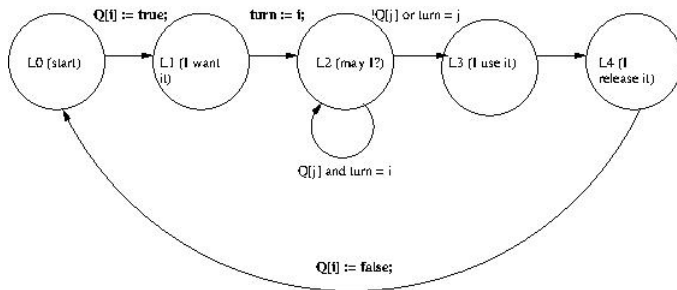
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What Is a *State*: Example

- We have two identical processes accessing to a shared resource
 - in the figure below, i, j denote the two processes
 - the well-known Peterson algorithm is used



What Is a *State*: Example

- The 5 “states” in the preceding figure are actually *modalities*
- From a model checking point of view, they correspond to *multiple* states
- To see which are the actual states, let us model this system with the following variables:
 - m_i , with $i = 1, 2$: the modality for process i
 - Q_i , with $i = 1, 2$: Q_i is a boolean which holds iff process i wants to access the shared resource
 - turn : shared variable



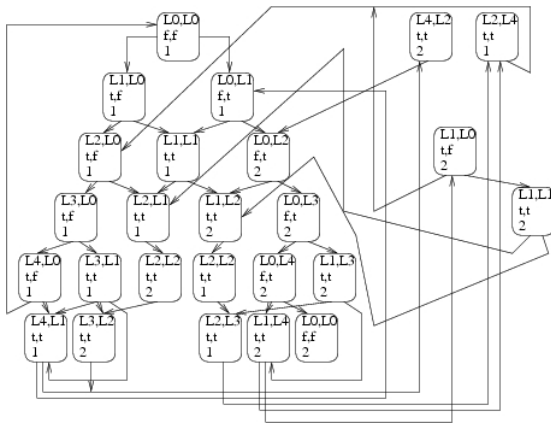
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What Is a *State*: Example

- Thus, the resulting model checking states are the following:



What Is a *State*: Example

- There are 25 *reachable states*
 - assuming state $\langle L0, L0, f, f, 1 \rangle$ as the starting one
- All *possible* states are 200
 - there are 3 variables with two possible values (the 2 variables Q, plus the turn variable) and 2 variables (P) with 5 possible values, thus $2^3 \times 5^2$ overall assignments
- The L0 modality for the first process encloses 6 (reachable) states

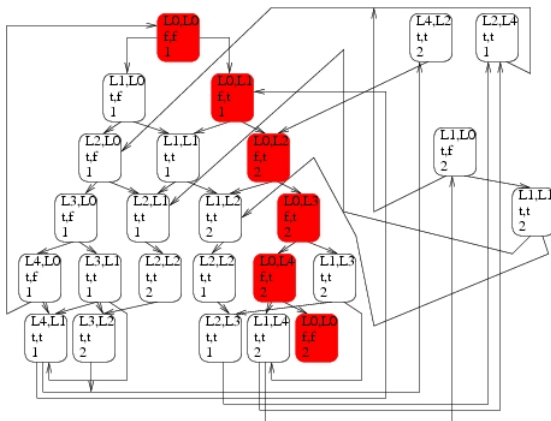


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What Is a *State*: Example



What Is a *State*: Example

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- The L0 modality for the first process encloses 6 (reachable) states
- No need of guards on transitions!



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From State Diagrams to Model Checking

- The UML-like state diagram is often useful to write the model
 - as we will see, this will depend on the model checker *input language*
- It is the model checker task to extract the global (reachable) graph as seen before
- And then analyze it



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Is Model Checking Important?

- ESA, NASA e IEC require most of their project to be model checked
- Important companies have dedicated laboratories for Model Checking
 - hardware: Intel, IBM, SUN, NVIDIA
 - software: IBM, SUN, Microsoft
- Many universities have research groups
 - USA: MIT, CMU, Austin, Stanford...
 - very close collaboration with companies
- The 3 “inventors” of Model Checking received Touring Award in 2007:
 - E. A. Emerson, E. M. Clarke, J. Sifakis

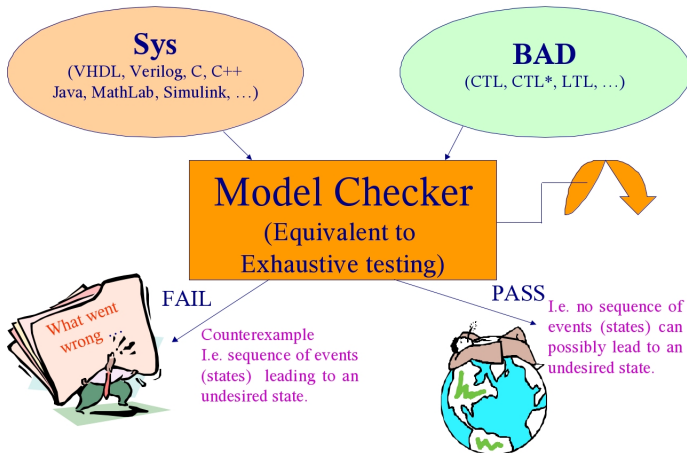


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Model Checking Usage



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Model Checking Usage

3 steps:

- 0 Choose the model checker M which is most suitable to the SUV $\mathcal{S}$ (and the property φ)
- 1 Describe $\mathcal{S}$ in the input language of M
- 2 Describe the property φ
- 3 Invoke the model checker and wait for the answer
 - OK $\Rightarrow \mathcal{S} \models \varphi$
 - FAIL $\Rightarrow$ counterexample
 - correct the error (it may happen that $\mathcal{S}$ or φ must be corrected instead...) and go back to step 3
 - OutOfMem or OutOfTime
 - adjust system parameters (or the description of $\mathcal{S}$)



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Model Checking Usage

- Not actually to verify programs with “standard” input and outputs
 - input is known in advance, e.g. in sorting; standalone computation
 - for such systems, testing can be complemented with theorem proving (or with manual proof derivation)
 - of course, budget must be taken into account
- Most used for *reactive systems*
 - always executing: monitoring (warns if something bad happens) or controlling (avoids that something bad happens)



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Model Checking: Pro and Cons

Pro

- Same guarantees of proof checking
- But requiring less “mathematics” and “computer science” knowledge

Cons

- Computational Complexity
 - causing “OutOfMem” and “OutOfTime”: *State Explosion Problem*
- You check a model of the system, not the actual system
 - though in some cases models can be automatically extracted from the system



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State Explosion Problem: Why?

- With some simplification, all Model Checking algorithms are essentially like this:
 - 1 Extract, from the description of the SUV $\mathcal{S}$, the *transition relation* of $\mathcal{S}$
 - 2 Compute the *reachable states* (*reachability*)
 - 3 Check if φ holds in all reachable states
- All steps may be computationally heavy, but let us focus on the reachability
 - see mutual exclusion example
- If $\mathcal{S}$ is described by n (binary) variables, then the number of reachable states is $O(2^n)$



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State Explosion Problem: Why?

- Such complexity cannot be avoided in the most general case
- Theoretically speaking, (LTL) Model Checking is P-SPACE complete
 - CTL Model Checking is in P, but as we will see this does not make things better
- There are different model checking algorithms, depending on the “type” of $\mathcal{S}$
 - each checker has its “preferred” SUVs



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Model Checking Algorithms

There are 3 categories:

- Explicit
- Implicit (symbolic)
- SAT-based



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 - each reachable state is separately stored
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Model Checking Algorithms

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 - many problems may be rewritten as SAT, but in model checking this works pretty well also in practice
 - software model checking



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Model Checking Algorithms

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 - very good for digital hardware
- SAT-based
 - many problems may be rewritten as SAT, but in model checking this works pretty well also in practice
 - software model checking
- Proof checker ibridations
 - not completely automatic, but better than proof checkers



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- Murphi or Mur φ , the simplest among “model checkers”
 - as all model checkers we will see in this course, Murphi may be freely downloaded with the source code, thus it may also be modified
 - links for download of all model checkers we will see are on the course web-page: https://igormelatti.github.io/sw_test_val/20232024/index.html



Murphi

- Formally, as all model checkers, Murphi needs the following input:
 - ① a description of the system S you want to verify (i.e., the “model” you want to “check”)
 - as we will see, this is essentially a Kripke structure
 - ② a property φ you want the system S to satisfy
- The output will be either OK or FAIL
 - if FAIL, it is possible to tell Murphi to print a *counterexample*



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Murphi

- In Murphi, both the description of $\mathcal{S}$ and of φ must be written in a single text file, following a precise syntax
 - in other model checkers we will see (e.g., SPIN), this syntax has a name; but this is not the case for Murphi
 - thus, we will refer to it simply as *Murphi input language*
 - as we will see, in many points Murphi input language is similar to some imperative programming languages, especially Pascal (for statements) and C (for expressions)



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- Murphi checks that all reachable states of S satisfy all invariants
 - a state $s \in S$ is *reachable* if there exists a path in the transition graph from an initial state to s
 - that is: starting from an initial state, there exists a chain of rules, each applied to the state obtained from the preceding one, leading to s
 - this is a *safety* property



- Example: G. L. Peterson protocol for mutual exclusion of 2 processes (1981)

```
boolean flag [2];
int turn;
void P0()
{
    while (true) {
        flag [0] = true;
        turn = 1;
        while (flag [1] && turn == 1) /* do nothing */;
        /* critical section */;
        flag [0] = false;
        /* remainder */;
    }
}
void P1()
{
    while (true) {
        flag [1] = true;
        turn = 0;
        while (flag [0] && turn == 0) /* do nothing */;
        /* critical section */;
        flag [1] = false;
        /* remainder */;
    }
}
void main()
{
    flag [0] = false;
    flag [1] = false;
    parbegin (P0, P1);
}
```

Peterson's Algorithm



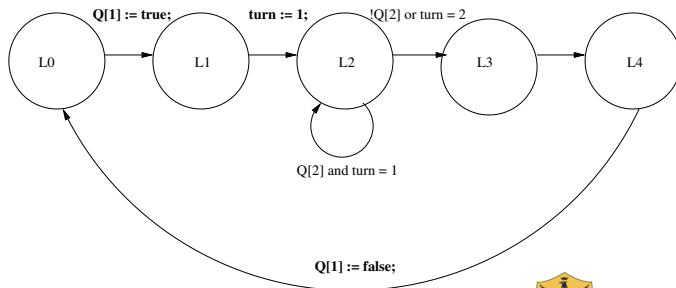
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Murphi

- Example: G. L. Peterson protocol for mutual exclusion of 2 processes (1981)
- UML-like state diagram: this is the first process; the second may be obtained exchanging 1's with 2's and viceversa



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- Example: G. L. Peterson protocol for mutual exclusion of 2 processes (1981)
 - two identical processes
 - each applies Peterson protocol to access to the critical section L3
 - the first issuing the request enters L3
 - Q is a global variable, defined as an array of two integers
 - each process i may modify $Q[i]$ and read $Q[(i + 1) \bmod 2]$
 - $turn$ is another global variable, which may be both read and modified by both processes



Murphi

- Murphi description for Peterson protocol: let's start with the variables
 - of course turn and Q, but also two variables P for the modality ("states" in the UML-like state diagram)
 - see `01.2_peterson.no_rulesets.no_parametric.m`
 - to this aim, we define constants and types
 - the N constant (number of processes) is here fictitious: only 2 processes, not more
 - this version of Peterson protocol only works for 2 processes
- thus, the state space is
$$S = \text{label_t}^2 \times \{\text{true}, \text{false}\}^2 \times \{1, 2\}$$



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Variables for Murphi Model Describing Peterson Protocol

P $v \in \{L0, L1, L2, L3, L4\}$ $v \in \{L0, L1, L2, L3, L4\}$

Q $v \in \{true, false\}$ $v \in \{true, false\}$

turn $v \in \{1..N\}$



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- Hence, $|S| = 5^2 \times 2^2 \times 2 = 200$ (there are 200 possible states)
 - as a matter of comparison, the “state” L0 in the UML-like state diagram actually contains $5^1 \times 2^2 \times 2 = 40$ states...
- However, as we will see, *reachable* states are about 10 times less
- 2 initial states: turn may be initialized with any value in its domain
- Note that `01.2_peterson.no_rulesets.no_parametric.m` we have rules repeated 2 times in a nearly equal fashion
- This can be done in this very simple model, but in general descriptions must be *parametric*



Murphi

- If we want to check Peterson with 3 processes, currently we would have to add rules in the description
 - very similar to the ones already present, only changing the index to 3
- Instead, it must be possible to only change the value of N from 2 to 3
- To write parametric descriptions in Murphi, rules are grouped with *rulesets*
 - an index will allow to describe the behavior of the generic process i
 - see `02.2_peterson.with_rulesets.no_parametric.m`, but invariant is still for two processes only



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- Finally, in `03.2.peterson.with_rulesets.parametric.m` also the invariant is parametric in N
 - `Exists  $x:T$   $E(x)$  End` is equivalent to $\bigvee_{x \in T} E(x)$
 - `Forall  $x:T$   $E(x)$  End` is equivalent to $\bigwedge_{x \in T} E(x)$
 - all types $T = \{x_1, \dots, x_{|T|}\}$ are finite, thus it is a finite formula



Kripke Structures

- Let AP be a set of “atomic propositions”
 - in the sense of first-order logic: each atomic proposition is either true or false
 - typically identified with lower case letters $p, q, \dots$
- A *Kripke Structure* (KS) over AP is a 4-tuple $\langle S, I, R, L \rangle$
 - S is a finite set, its elements are called *states*
 - $I \subseteq S$ is a set of *initial states*
 - $R \subseteq S \times S$ is a *transition relation*
 - $L : S \rightarrow 2^{AP}$ is a *labeling function*



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Labeled Transition Systems

- A *Labeled Transition System* (LTS) is a 4-tuple $\langle S, I, \Lambda, \delta \rangle$
 - S is a finite set of states as before
 - $I \subseteq S$ is a set of initial states as before (not always included)
 - Λ is a finite set of *labels*
 - $\delta \subseteq S \times \Lambda \times S$ is a *labeled transition relation*



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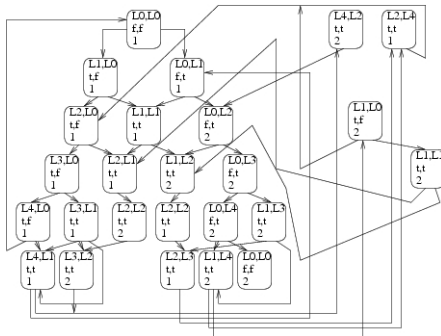
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Peterson's Mutual Exclusion as a Kripke Structure

- $S = \{(p_1, p_2, q_1, q_2, t) \mid p_1, p_2 \in \{L0, L1, L2, L3, L4\}, q_1, q_2 \in \{0, 1\}, t \in \{1, 2\}\} = \{L0, L1, L2, L3, L4\}^2 \times \{0, 1\}^2 \times \{1, 2\}$
- $I = \{L0\}^2 \times \{0\}^2 \times \{1, 2\}$
- R : see next slide
- $AP = \{(P_1 = v) \mid v \in \{L0, L1, L2, L3, L4\}\} \cup \{(P_2 = v) \mid v \in \{L0, L1, L2, L3, L4\}\} \cup \{(Q_1 = v) \mid v \in \{0, 1\}\} \cup \{(Q_2 = v) \mid v \in \{0, 1\}\} \cup \{(\text{turn} = v) \mid v \in \{1, 2\}\}$
 - e.g.: $L(L0, L0, 0, 0, 1) = \{(P_1 = L0), (P_2 = L0), (Q_1 = 0), (Q_2 = 0), (\text{turn} = 1)\}$



Peterson's Mutual Exclusion as a Kripke Structure



E.g.: $((L0, L0, 0, 0, 1), (L1, L0, 1, 0, 1)) \in R$, whilst
 $((L0, L0, 0, 0, 1), (L2, L0, 0, 0, 1)) \notin R$

Transitions in R corresponds to arrows in the figure above



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Kripke Structure vs Labeled Transition Systems

- KSs have atomic propositions on states, LTSs have labels on transitions
- In model checking, atomic propositions are mandatory
 - to specify the formula to be verified, as we will see
 - a first example was the invariant in Murphi
- Instead, it is not required to have a label on transitions
 - Murphi allows to do so, but it is optional
 - may be easily added automatically, if needed
- Labels are typically needed when:
 - we deal with macrostates, as in UML state diagrams
 - when we are describing a complex system by specifying its sub-components, so labels are used for synchronization



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Total Transition Relation

- In many cases, the transition relation R is required to be *total*
- $\forall s \in S. \exists s' \in S : (s, s') \in R$
 - this of course allows also $s = s'$ (*self loop*)
- In the Peterson's example, the relation is actually total
 - Murphi allows also non-total relations, by using option `-ndl`
 - note however that not giving option `-ndl` is stronger:
 $\forall s \in S. \exists s' \in S : s \neq s' \wedge (s, s') \in R$
 - otherwise, if s is s.t. $\forall s'. s = s' \vee (s, s') \notin R$, Murphi calls s a *deadlock* state
 - that is, you cannot go anywhere, except possibly self looping on s
- By deleting any rule, we will obtain a non-total transition relation



Non-Determinism

- The transition relation is, as the name suggests, a relation
- Thus, starting from a given state, it is possible to go to many different states
 - in a deterministic system,
$$\forall s_1, s_2, s_3 \in S. (s_1, s_2) \in R \wedge (s_1, s_3) \in R \rightarrow s_2 = s_3$$
 - this does not hold for KSs
- This means that, starting from state s_1 , the system may *non-deterministically* go either to s_2 or to s_3
 - or many other states
- Motivations for non-determinism: modeling choices!
 - underspecified subsystems
 - unpredictable interleaving
 - interactions with an uncontrollable environment
 - ...



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Some Useful Notation

- Given a KS $\mathcal{S} = \langle S, I, R, L \rangle$, we can define:
 - the *predecessor* function $\text{Pre}_{\mathcal{S}} : S \rightarrow 2^S$
 - defined as $\text{Pre}_{\mathcal{S}}(s) = \{s' \in S \mid (s', s) \in R\}$
 - we will write simply $\text{Pre}(s)$ when $\mathcal{S}$ is understood
 - the *successor* function $\text{Post} : S \rightarrow 2^S$
 - defined as $\text{Post}(s) = \{s' \in S \mid (s, s') \in R\}$
- Note that, if $\mathcal{S}$ is deterministic, $\forall s \in S. |\text{Post}(s)| \leq 1$



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Paths in KSs

- A path (or *execution*) on a KS $\mathcal{S} = \langle S, I, R, L \rangle$ is a sequence $\pi = s_0 s_1 s_2 \dots$ such that:
 - $\forall i \geq 0. s_i \in S$ (it is composed by states)
 - $\forall i \geq 0. (s_i, s_{i+1}) \in R$ (it only uses valid transitions)
- We will denote i -th state of a path as $\pi(i) = s_i$
- Note that paths in LTSs also have actions: $\pi = s_0 a_0 s_1 a_1 \dots$
s.t. $(s_i, a_i, s_{i+1}) \in \delta$



Paths in KSs

- The *length* of a path π is the number of states in π
 - paths can be either finite $\pi = s_0 s_1 \dots s_n$, in which case $|\pi| = n + 1$
 - or infinite $\pi = s_0 s_1 \dots$, in which case $|\pi| = \infty$
- We will denote the prefix of a path up to i as $\pi|_i = s_0 \dots s_i$
 - a prefix of a path is always a finite path
- A path π is *maximal* iff one of the following holds
 - $|\pi| = \infty$
 - $|\pi| = n + 1$ and $|\text{Post}(\pi(n))| = 0$
 - that is, $\forall s \in S. (\pi(n), s) \notin R$
 - i.e., the last state of the path has no successors
 - often called *terminal state*
- If R is total, maximal paths are always infinite
 - for many model checking algorithms, this is required



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Reachability

- The set of paths of $\mathcal{S}$ starting from $s \in S$ is denoted by $\text{Path}(\mathcal{S}, s) = \{\pi \mid \pi \text{ is a path in } \mathcal{S} \wedge \pi(0) = s\}$
- The set of paths of $\mathcal{S}$ is denoted by $\text{Path}(\mathcal{S}) = \cup_{s \in I} \text{Path}(\mathcal{S}, s)$
 - that is, they must start from an initial state
- A state $s \in S$ is *reachable* iff $\exists \pi \in \text{Path}(\mathcal{S}), k < |\pi| : \pi(k) = s$
 - i.e., there exists a path from an initial state leading to s through valid transitions
- The set of reachable states is defined by $\text{Reach}(\mathcal{S}) = \{\pi(i) \mid \pi \in \text{Path}(\mathcal{S}), i < |\pi|\}$



Safety Property Verification

- Verification of *invariants*: nothing bad happens
- The property is a formula $\varphi : S \rightarrow \{0, 1\}$
 - built using boolean combinations of atomic propositions in $p \in AP$
 - i.e., the syntax is

$$\Phi ::= (\Phi) \mid \Phi \wedge \Phi \mid \Phi \vee \Phi \mid \neg \Phi \mid p$$

- The KS $\mathcal{S}$ satisfies φ iff φ holds on all reachable states
 - $\forall s \in \text{Reach}(\mathcal{S}). \varphi(s) = 1$
- Note that it may happen that $\varphi(s) = 0$ for some $s \in S$: never mind, if $s \notin \text{Reach}(\mathcal{S})$



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How to Verify a Murphi Description $\mathcal{M}$

- Theoretically, extract KS $\mathcal{S}$ and property φ from $\mathcal{M}$ as described above
 - for a given invariant I in $\mathcal{M}$, $\varphi(s) = \zeta(I, s)$ for all $s \in S$
- Then, KS $\mathcal{S}$ satisfies φ iff φ holds on all reachable states
 - $\forall s \in \text{Reach}(\mathcal{S}). \varphi(s) = 1$
- Thus, consider KS as a graph and perform a visit
 - states are nodes, transitions are edges
- If a state e s.t. $\varphi(e) = 0$ is found, then we have an error
- Otherwise, all is ok



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How to Verify a Murphi Description $\mathcal{M}$

- From a practical point of view, many optimization may be done, but let us stick to the previous scheme
- The worst case time complexity for a DFS or a BFS is $O(|V| + |E|)$ (and same for space complexity)
- For KSs, this means $O(|S| + |R|)$, thus it is linear in the size of the KS
- Is this good? NO! Because of the *state space explosion problem*
- Assuming that B bits are needed to encode each state
 - i.e., $B = \sum_{i=1}^n b_i$, being b_i the number of bits to encode domain D_i
- We have that $|S| = O(2^B)$



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State Space Explosion

- The “practical” input dimension is B , rather than $|S|$ or $|R|$
- Typically, for a system with N components, we have $O(N)$ variables, thus $O(B)$ encoding bits
- It is very common to verify a system with N components, and then (if N is ok) also for $N + 1$ components
 - verifying a system with a generic number N of components is a proof checker task...
- This entails an exponential increase in the size of $|S|$
- Thus we need “clever” versions of BFS/DFS



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Standard BFS: No Good for Model Checking

- Assumes that all graph nodes are in RAM
- For KSs, graph nodes are states, and we know there are too many
 - state space explosion
- You also need a full representation of the graph, thus also edges must be in RAM
 - using adjacency matrices or lists does not change much
 - for real-world systems, you may easily need TB of RAM
- Even if you have all the needed RAM, there is a huge preprocessing time needed to build the graph from the Murphi specification
- Then, also BFS itself may take a long time



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Murphi BFS

- We need a definition inbetween the model and the KS: NFSS (Nondeterministic Finite State System)
- $\mathcal{N} = \langle S, I, \text{Post} \rangle$, plus the invariant φ
 - S is the set of states, $I \subseteq S$ the set of initial states
 - $\text{Post} : S \rightarrow 2^S$ is the successor function as defined before
 - given a state s , it returns T s.t. $t \in T \rightarrow (s, t) \in R$
 - no labeling, we already have φ



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Murphi BFS

- KSs and NFSSs differ on having Post instead of R
- Post may easily be defined from the Murphi specification
- Such definition is implicit, as programming code, thus avoiding to store adjacency matrices or lists
 - $t \in \text{Post}(s)$ iff there is a rule $T_i \in T$ s.t. T_i guard is true in s and T_i body changes s to t
 - see above for using η and ζ
 - Essentially, if the current state is s , it is sufficient to inspect all (flattened) rules in the Murphi specification $\mathcal{M}$
 - for all guards which are enabled in s , execute the body so as to obtain t , and add t to $\text{next}(s)$
 - This is done “on the fly”, only for those states s which must be explored



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Murphi Simulation

```
void Make_a_run(NFSS  $\mathcal{N}$ , invariant  $\varphi$ )
{
  let  $\mathcal{N} = \langle S, I, \text{Post} \rangle$ ;
  s_curr = pick_a_state(I);
  if (! $\varphi(s_{\text{curr}}$ ))
    return with error message;
  while (1) { /* loop forever */
    s_next = pick_a_state(Post(s_curr));
    if (! $\varphi(s_{\text{next}}$ ))
      return with error message;
    s_curr = s_next;
  }
}
```



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Murphi Simulation

```
void Make_a_run(NFSS  $\mathcal{N}$ , invariant  $\varphi$ )
{
  let  $\mathcal{N} = \langle S, I, \text{Post} \rangle$ ;
  s_curr = pick_a_state(I);
  if ( $\neg \varphi(s_{\text{curr}})$ )
    return with error message;
  while (1) { /* loop forever */
    if ( $\text{Post}(s_{\text{curr}}) = \emptyset$ )
      return with deadlock message;
    s_next = pick_a_state( $\text{Post}(s_{\text{curr}})$ );
    if ( $\neg \varphi(s_{\text{next}})$ )
      return with error message;
    s_curr = s_next;
  }
}
```



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Murphi Simulation

- Similar to testing
- If an error is found, the system is bugged
 - or the model is not faithful
 - actually, Murphi simulation is used to understand if the model itself contains errors
- If an error is not found, we cannot conclude anything
- The error state may lurk somewhere, out of reach for the random choice in `pick_a_state`



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Standard BFS (Cormen-Leiserson-Rivest)

BFS(G, s)

```
1  for ogni vertice  $u \in V[G] - \{s\}$ 
2      do  $color[u] \leftarrow \text{WHITE}$ 
3          $d[u] \leftarrow \infty$ 
4          $\pi[u] \leftarrow \text{NIL}$ 
5   $color[s] \leftarrow \text{GRAY}$ 
6   $d[s] \leftarrow 0$ 
7   $\pi[s] \leftarrow \text{NIL}$ 
8   $Q \leftarrow \{s\}$ 
9  while  $Q \neq \emptyset$ 
10     do  $u \leftarrow head[Q]$ 
11        for ogni  $v \in Adj[u]$ 
12            do if  $color[v] = \text{WHITE}$ 
13                then  $color[v] \leftarrow \text{GRAY}$ 
14                    $d[v] \leftarrow d[u] + 1$ 
15                    $\pi[v] \leftarrow u$ 
16                   ENQUEUE( $Q, v$ )
17     DEQUEUE( $Q$ )
18   $color[u] \leftarrow \text{BLACK}$ 
```



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Murphi BFS

```
FIFO_Queue Q;  
HashTable T;  
  
bool BFS(NFSS  $\mathcal{N}$ , AP  $\varphi$ )  
{  
  let  $\mathcal{N} = (S, I, \text{Post})$ ;  
  foreach s in I {  
    if (! $\varphi(s)$ )  
      return false;  
  }  
  foreach s in I  
    Enqueue(Q, s);  
  foreach s in I  
    HashInsert(T, s);
```



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Murphi BFS

```
while (Q  $\neq$   $\emptyset$ ) {  
  s = Dequeue(Q);  
  foreach s_next in Post(s) {  
    if (! $\varphi$ (s_next))  
      return false;  
    if (s_next is not in T) {  
      Enqueue(Q, s_next);  
      HashInsert(T, s_next);  
    } /* if */ } /* foreach */ } /* while */  
return true;  
}
```



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Murphi BFS

- Edges are never stored in memory
- (Reachable) states are stored in memory only at the end of the visit
 - inside hashtable T
- This is called *on-the-fly* verification
- States are marked as visited by putting them inside an hashtable
 - rather than coloring them as gray or black
 - which needs the graph to be already in memory



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State Space Explosion

- State space explosion hits in the FIFO queue Q and in the hashtable T
 - and of course in running time...
- However, Q is not really a problem
 - it is accessed *sequentially*
 - always in the front for extraction, always in the rear for insertion
 - can be efficiently stored using disk, much more capable of RAM
- T is the real problem
 - random access, not suitable for a file
 - what to do?
 - before answering, let's have a look at Murphi code



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Murphi Usage

- As for all *explicit* model checker, a Murphi verification has the following steps:
 - 0 compile Murph source code and write a Murphi model `model.m`
 - 1 invoke Murphi compiler on `model.m`: this generates a file `model.cpp`
 - `mu options model.m`
 - see `mu -h` for available options
 - 2 invoke C++ compiler on `model.cpp`: this generates an executable file
 - `g++ -Ipath_to_include model.cpp -o model`
 - `path_to_include` is the include directory inside Murphi distribution
 - 3 invoke the executable file
 - `./model options`
 - see `./model -h` for available options



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Beyond Invariants

- Invariants represent a huge share of properties to be verified on a system
- For many systems, one may be happy with invariants only
 - “nothing bad happens”, that’s all folks
- However, it is not always sufficient: a non-running system of course satisfies invariants
 - no starting states, thus no reachable states...



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Safety vs. Liveness

- **Safety** properties: something bad must never happen
 - example: in the Peterson's protocol, it must not happen that both processes are accessing the resource (L3 in the Murphi model)
- Invariants are a special case of safety properties
 - there are some safety properties which are not invariants
 - however, they can be expressed with invariants by adding variables to the Kripke Structure
 - in the following, we will consider "invariants" and "safety properties" as synonyms
- **Liveness** properties: something good will eventually happen
 - example: in the Peterson's protocol, both processes will eventually access the resource
 - not at the same time!
 - cannot be expressed with invariants



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Safety vs. Liveness

- Notation: let $\mathcal{S}$ be a KS and φ be a formula in any logic
- $\mathcal{S} \models \varphi$ is true iff φ is true in $\mathcal{S}$
 - what this means depends on the logic, as we will see
- For most properties φ , if $\mathcal{S} \not\models \varphi$ then there exists a path $\pi \in \text{Path}(\mathcal{S})$ which is a *counterexample*
- For safety properties, $|\pi| < \infty$
 - $\mathcal{S}$ arrives to an *unsafe* state and that's it
- For liveness properties, $|\pi| = \infty$
 - since $\mathcal{S}$ is finite, this implies that π contains a loop (*lasso*) in its final part



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Safety vs. Liveness

- Equivalent definition for a safety formula: given a finite counterexample, every extension still contains the error
- There is one formula which is both safety and liveness: the true invariant
 - it cannot have a counterexample...
- There are formulas which are neither safety nor liveness
 - their counterexample is not a path
- For typically used formulas, they are either safety or liveness properties

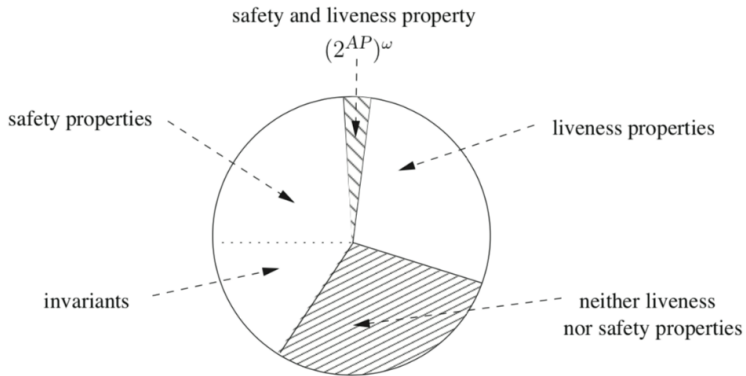


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If we identify a property by the set of its models ($\varphi = \{\sigma \mid \sigma \models \varphi\}$)



Model Checking Logics: Preliminaries

- Model Checking logics are based on the concept of *execution* of a Kripke structure $\mathcal{S}$
 - thus, on $\pi \in \text{Path}$
- Often, paths are directly viewed as a sequence of atomic propositions, rather than states
 - from $\pi = s_1, s_2, \dots$ to $AP(\pi) = L(s_1), L(s_2), \dots$
- Focusing on executions allows to model *time*
 - property on paths, especially useful for liveness properties
 - time in the sense that we have something coming before of something else (in a path...)
- Trade-off between
 - logics expressiveness: interesting properties can be written
 - logics efficiency: there is an efficient model checking algorithm to compute if $\mathcal{S} \models \varphi$



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Model Checking Logics: Preliminaries

- We will focus on the two leading Model Checking logics: LTL and CTL
 - with some hints on CTL*
 - LTL (Linear-time Temporal Logic) established by Pnueli in 1977
 - CTL (Computation Tree Logic) established by Clarke and Emerson in 1981
 - used for IEEE standards:
 - PSL (Property Specification Language, IEEE Standard 1850)
 - SVA (SystemVerilog Assertions, IEEE Standard 1800).
- We will see syntax and semantics of both logics
 - syntax: how a valid formula is written
 - semantics: what a valid formula “means”
 - that is, when $\mathcal{S} \models \varphi$ holds



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LTL Syntax

$$\Phi ::= p \mid \Phi_1 \wedge \Phi_2 \mid \neg \Phi \mid (\Phi) \mid \mathbf{X}\Phi \mid \Phi_1 \mathbf{U} \Phi_2$$

- Other derived operators:
 - of course true, false, OR and other propositional logic connectors
 - future (or eventually): $\mathbf{F}\Phi = \text{true} \mathbf{U} \Phi$
 - globally: $\mathbf{G}\Phi = \neg(\text{true} \mathbf{U} \neg\Phi)$
 - release: $\Phi_1 \mathbf{R} \Phi_2 = \neg(\neg\Phi_1 \mathbf{U} \neg\Phi_2)$
 - weak until: $\Phi_1 \mathbf{W} \Phi_2 = (\Phi_1 \mathbf{U} \Phi_2) \vee \mathbf{G}\Phi_1$
- Other notations:
 - next: $\mathbf{X}\Phi = \bigcirc\Phi$
 - $\mathbf{G}\Phi = \square\Phi$
 - $\mathbf{F}\Phi = \diamond\Phi$
- We are dropping *past operators*, thus this is *pure future LTL*



LTL Semantics

- Goal: formally defining when $\mathcal{S} \models \varphi$, being $\mathcal{S}$ a KS and φ an LTL formula
 - we say that $\mathcal{S}$ *satisfies* φ , or φ *holds in* $\mathcal{S}$
- This is true when, for all paths π of $\mathcal{S}$, π satisfies φ
 - i.e., $\forall \pi \in \text{Path}(\mathcal{S}). \pi \models \varphi$
 - symbol $\models$ is overloaded...
- For a given π , $\pi \models \varphi$ iff $\pi, 0 \models \varphi$
- Finally, to define when $\pi, i \models \varphi$, a recursive definition over the recursive syntax of LTL is provided
 - $\pi \in \text{Path}(\mathcal{S}), i \in \mathbb{N}$



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LTL Semantics for $\pi, i \models \varphi$

- $\forall \pi \in \text{Path}(\mathcal{S}), i \in \mathbb{N}. \pi, i \models \text{true}$
- $\pi, i \models p$ iff $p \in L(\pi(i))$
- $\pi, i \models \Phi_1 \wedge \Phi_2$ iff $\pi, i \models \Phi_1 \wedge \pi, i \models \Phi_2$
- $\pi, i \models \neg \Phi$ iff $\pi, i \not\models \Phi$
- $\pi, i \models \mathbf{X}\Phi$ iff $\pi, i + 1 \models \Phi$
- $\pi, i \models \Phi_1 \mathbf{U} \Phi_2$ iff $\exists k \geq i: \pi, k \models \Phi_2 \wedge \forall i \leq j < k. \pi, j \models \Phi_1$



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LTL Semantics for Added Operators

- It is easy to prove that:
 - $\pi, i \models \mathbf{G}\Phi$ iff $\forall j \geq i. \pi, j \models \Phi$
 - $\pi, i \models \mathbf{F}\Phi$ iff $\exists j \geq i. \pi, j \models \Phi$
 - $\pi, i \models \Phi_1 \mathbf{R} \Phi_2$ iff $\forall k \geq i. \pi, k \models \Phi_2 \vee \exists i \leq j < k : \pi, j \models \Phi_1$
 - i.e., $\forall k \geq i. \pi, k \not\models \Phi_2 \rightarrow \exists i \leq j < k : \pi, j \models \Phi_1$
 - i.e., $\forall k \geq i. \forall i \leq j < k. \pi, j \not\models \Phi_1 \rightarrow \pi, k \models \Phi_2$
 - $\pi, i \models \Phi_1 \mathbf{W} \Phi_2$ iff $(\forall j \geq i. \pi, j \models \Phi_1) \vee (\exists k \geq i : \pi, k \models \Phi_2 \wedge \forall i \leq j < k. \pi, j \models \Phi_1)$
- For many formulas, it is silently required that paths are infinite
- That's why transition relations in KSs must be total



LTL Semantics: Typical Paths for Common Formulas

- Let us say that, for $p \in AP$, $p \in \{P \in 2^{AP} \mid p \in P\}$
 - that is, p is any subset of atomic propositions containing p
 - $\{p\}, \{p, q\}, \{p, r, s\} \dots$
 - furthermore, $\bar{p} = \neg p \in \{P \in 2^{AP} \mid p \notin P\}$
 - $\{q\}, \{q, r\}, \{r, s\} \dots$
 - finally, $\perp$ denotes any subset of atomic propositions
- If $\pi \models \mathbf{G}p$, then $\pi = p^\omega$
 - of course, this includes, e.g., $\pi = \{p, q\}\{p, r\}\{p\}\{p, q\}\{p\} \dots$
 - $\pi, 3 \models \mathbf{G}p$: $\pi = \perp \perp \perp p^\omega$
- If $\pi \models \mathbf{F}p$, then $\pi = \perp^* p \perp^\omega$
- If $\pi \models p \mathbf{U} q$, then $\pi = \{p, \bar{q}\}^* q \perp^\omega$
- If $\pi \models p \mathbf{W} q$, then either $\pi = \{p, \bar{q}\}^* q \perp^\omega$ or $\pi = p^\omega$
- If $\pi \models p \mathbf{R} q$, then either $\pi = q^\omega$ or $\pi = \{\bar{p}, q\}^* \{p, q\} \perp^\omega$
 - q must be kept holding till when a p appears and releases $q \dots$



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Safety and Liveness Properties in LTL

- Given an LTL formula φ , φ is a safety formula iff
$$\forall \mathcal{S}. (\exists \pi \in \text{Path}(\mathcal{S}) : \pi \not\models \varphi) \rightarrow \exists k : \pi|_k \not\models \varphi$$
- Given an LTL formula φ , φ is a liveness formula iff
$$\forall \mathcal{S}. (\exists \pi \in \text{Path}(\mathcal{S}) : \pi \not\models \varphi) \rightarrow |\pi| = \infty$$
- All LTL formulas are either safety, liveness, or the AND of a safety and a liveness
 - being defined on paths, the counterexample is always a path
- Safety properties are those involving only **G**, **X**, true and atomic propositions
- Liveness are all those involving an **F**, or a **U** where the first formula is not the constant true
- Some formulas are both safety and liveness, like true, **G** true and so on

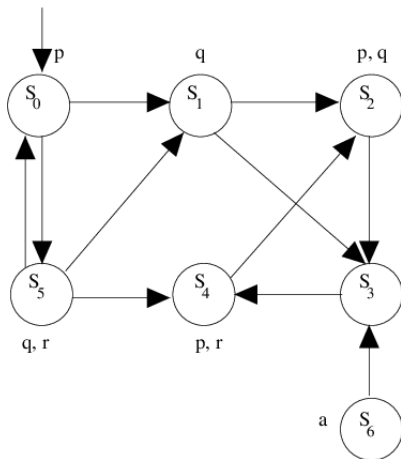


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LTL Examples



$\mathcal{S} \models \mathbf{F}p$ since p holds in the first state

For full: let $\pi \in \text{Path}(\mathcal{S})$

$\pi, 0 \models \mathbf{F}p$ with $j = 0$

recall: $\pi, i \models \mathbf{F}\Phi$ iff

$\exists j \geq i. \pi, j \models \Phi$

$\pi, i \models p$ iff $p \in L(\pi(i))$

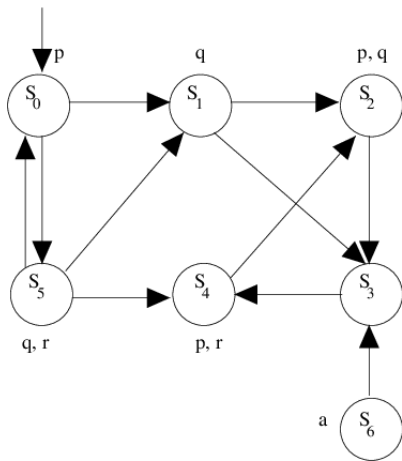


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LTl Examples



$\mathcal{S} \not\models \mathbf{F}a$ since s_6 is not reachable from s_0

counterexample: $\pi = s_0 s_5 s_0 s_5 \dots$

For full: $\pi, 0 \not\models \mathbf{F}a$ as, for all $j \geq 0$, $a \notin L(\pi(j))$

Counterexample is infinite, thus this is a liveness property
Any finite prefix of π is not a counterexample

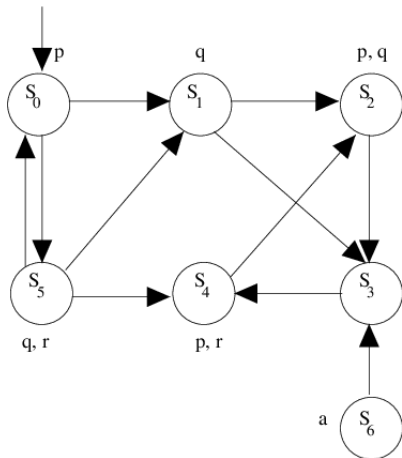


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LTL Examples



$\mathcal{S} \not\models \mathbf{G}p$ since there are many counterexamples, here is one:

$\pi = s_0 s_5 s_0 s_5 \dots$

For full: $\pi, 0 \not\models \mathbf{G}p$ with $j = 1$

recall: $\pi, i \models \mathbf{G}\Phi$ iff

$\forall j \geq i. \pi, j \models \Phi$

$\pi, i \models p$ iff $p \in L(\pi(i))$

Safety property, actually $\pi|_2$ is enough

Every path having $\pi|_2$ as a prefix is a counterexample

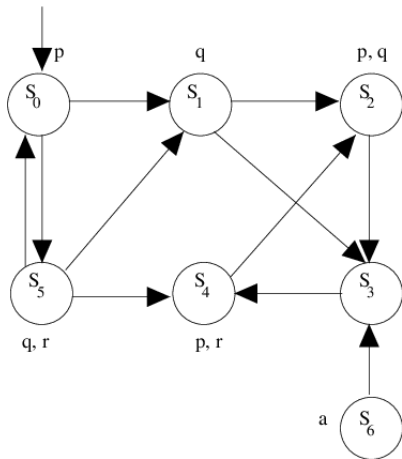


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LTL Examples



$\mathcal{S} \models \mathbf{G}\neg a$ since s_6 is not reachable from s_0

For full: let $\pi \in \text{Path}(\mathcal{S})$
 $\pi, 0 \models \mathbf{G}\neg a$ as the only state s with $a \in L(s)$ is s_6 , which is not reachable from s_0

recall: $\pi \in \text{Path}(\mathcal{S})$ implies $\pi(0) \in I$, thus $\pi(0) = s_0$ here

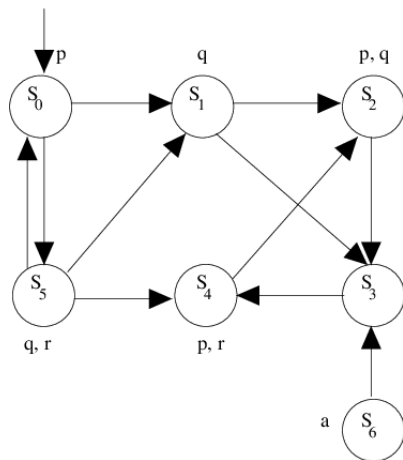


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LTL Examples



$\mathcal{S} \models p \text{ U } q$ since $p \in L(s_0)$,
 $\text{next}(s_0) = \{s_1, s_5\}$ and $q \in L(s_1) \wedge q \in L(s_5)$

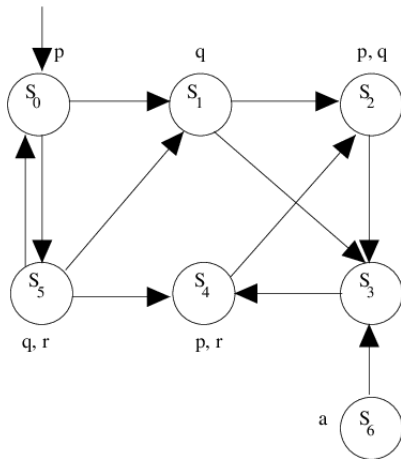


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LTL Examples



$\mathcal{S} \not\models p \mathbf{U} r$, a counterexample is $\pi = s_0 s_1 (s_2 s_3 s_4)$

Again this is a liveness formula, even if $\pi|_1$ would have been enough

In fact, you have to rule out $\{p, \bar{r}\}^\omega \dots$

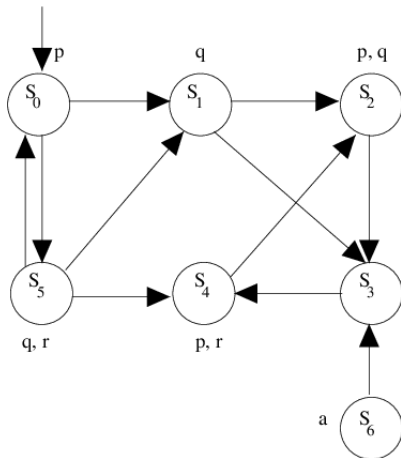


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LTl Examples



$\mathcal{S} \not\models \neg(p \mathbf{U} r)$, a counterexample is $\pi = (s_0 s_5)$

In fact, $(s_0 s_5), 0 \models p \mathbf{U} r$

Thus it may happen that $\mathcal{S} \not\models \Phi$ and $\mathcal{S} \not\models \neg(\Phi)$

Instead, it is impossible that $\mathcal{S} \models \Phi$ and $\mathcal{S} \models \neg(\Phi)$

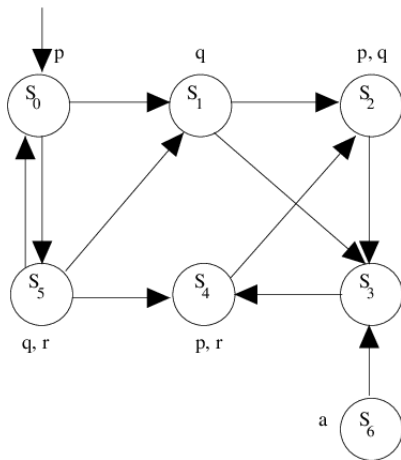


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LTL Examples



$\mathcal{S} \not\models \mathbf{FG}p$, a counterexample is
 $\pi = s_0 s_1 (s_2 s_3 s_4)$
Again this is a liveness formula

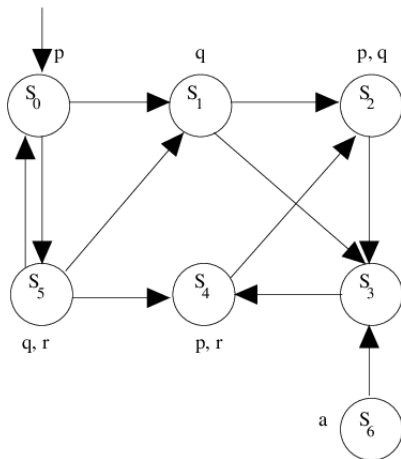


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LTL Examples



$\mathcal{S} \models \mathbf{GF}p$

All lassos are s_0s_5 or $s_2s_3s_4$

In both such lassos, there are states in which p holds

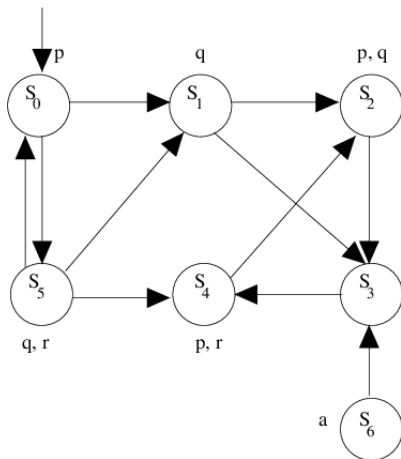


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LTl Examples



$\mathcal{S} \models \mathbf{GF}p \vee \mathbf{FG}p$

Consequence of the two previous slides

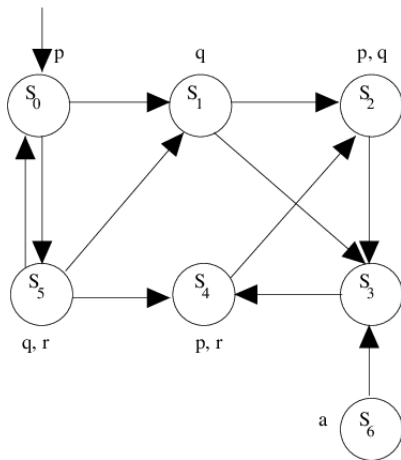


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LTl Examples



$\mathcal{S} \not\models \mathbf{G}(p \mathbf{U} q)$, a counterexample is $\pi = s_0 s_1 (s_2 s_3 s_4)$
 $(p \mathbf{U} q)$ must hold at any reachable state
Ok in s_0, s_1, s_2 , but not in s_3



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LTL Non-Toy Examples

- Recall the Peterson's protocol: checking mutual exclusion is $\mathbf{G}(\neg(p \wedge q))$, being $p = P[1] = L3$, $q = P[2] = L3$
 - all invariants are of the form $\mathbf{G}P$, where P does not contain modal operators $\mathbf{X}$, $\mathbf{U}$ or $\mathbf{F}$
- Checking that both processes access to the critical section *infinitely often* is $\mathbf{GF} P[1] = L3 \wedge \mathbf{GF} P[2] = L3$
 - liveness property: no process is infinitely banned to access the critical section
- Even better: $\mathbf{G} (P[1] = L2 \rightarrow \mathbf{F} P[1] = L3)$
 - the same for the other process
 - since it is symmetric, this is actually enough



Equivalence Between LTL Properties

- Definition of equivalence between LTL properties:
 $\varphi_1 \equiv \varphi_2 \text{ iff } \forall \mathcal{S}. \mathcal{S} \models \varphi_1 \Leftrightarrow \mathcal{S} \models \varphi_2$
 - equivalent: $\forall \sigma \dots$
- Idempotency:
 - $\mathbf{FF}p \equiv \mathbf{F}p$
 - $\mathbf{GG}p \equiv \mathbf{G}p$
 - $p \mathbf{U} (p \mathbf{U} q) \equiv (p \mathbf{U} q) \mathbf{U} q \equiv p \mathbf{U} q$
- Absorption:
 - $\mathbf{GFG}p \equiv \mathbf{FG}p$
 - $\mathbf{FGF}p \equiv \mathbf{GF}p$
- Expansion (used by LTL Model Checking algorithms!):
 - $p \mathbf{U} q \equiv q \vee (p \wedge \mathbf{X}(p \mathbf{U} q))$
 - $\mathbf{F}p \equiv p \vee \mathbf{XF}p$
 - $\mathbf{G}p \equiv p \wedge \mathbf{XG}p$



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CTL Syntax

$$\Phi ::= p \mid \Phi_1 \wedge \Phi_2 \mid \neg \Phi \mid (\Phi) \mid \mathbf{EX}\Phi \mid \mathbf{EG}\Phi \mid \mathbf{E}\Phi_1 \mathbf{U} \Phi_2$$

- Other derived operators (besides true, false, OR, etc):
 - $\mathbf{EF}\Phi = \mathbf{Etrue} \mathbf{U} \Phi$
 - cannot be defined using $\mathbf{E}\neg\mathbf{G}\neg\Phi$, as this is not a CTL formula
 - actually, it is a CTL* formula (see later)
 - $\mathbf{AF}\Phi = \neg\mathbf{EG}\neg\Phi$, $\mathbf{AG}\Phi = \neg\mathbf{EF}\neg\Phi$, $\mathbf{AX}\Phi = \neg\mathbf{EX}\neg\Phi$
 - $\mathbf{A}\Phi_1 \mathbf{U} \Phi_2 = (\neg\mathbf{E}\neg\Phi_2 \mathbf{U} (\neg\Phi_1 \wedge \neg\Phi_1)) \wedge \neg\mathbf{EG}\neg\Phi_2$
 - $\Phi_1 \mathbf{AU}\Phi_2 = \mathbf{A}\Phi_1 \mathbf{U}\Phi_2$, $\Phi_1 \mathbf{EU}\Phi_2 = \mathbf{E}\Phi_1 \mathbf{U}\Phi_2$



Comparison with LTL Syntax

$$\Phi ::= \text{true} \mid p \mid \Phi_1 \wedge \Phi_2 \mid \neg \Phi \mid (\Phi) \mid \mathbf{X}\Phi \mid \Phi_1 \mathbf{U} \Phi_2$$

- Essentially, all temporal operators are preceded by either **E** or **G**
 - with some care for **U**



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CTL Semantics

- Goal: formally defining when $\mathcal{S} \models \varphi$, being $\mathcal{S}$ a KS and φ a CTL formula
- This is true when, for all initial states $s \in I$ of $\mathcal{S}$, $s \models \varphi$
 - thus, CTL is made of *state* formulas
 - LTL has *path* formulas
- To define when $s \models \varphi$, a recursive definition over the recursive syntax of CTL is provided
 - no need of an additional integer as for LTL syntax



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CTL Semantics for $s \models \varphi$

- $\forall s \in S. s \models \text{true}$
- $s \models p$ iff $p \in L(s)$
- $s \models \Phi_1 \wedge \Phi_2$ iff $s \models \Phi_1 \wedge s \models \Phi_2$
- $s \models \neg\Phi$ iff $s \not\models \Phi$
- $s \models \mathbf{EX}\Phi$ iff $\exists \pi \in \text{Path}(S, s). \pi(1) \models \Phi$
- $s \models \mathbf{EG}\Phi$ iff $\exists \pi \in \text{Path}(S, s). \forall j. \pi(j) \models \Phi$
- $s \models \mathbf{E}\Phi_1 \mathbf{U} \Phi_2$ iff
 $\exists \pi \in \text{Path}(S, s) \exists k : \pi(k) \models \Phi_2 \wedge \forall j < k. \pi(j) \models \Phi_1$



CTL Semantics for Added Operators

- It is easy to prove that:
 - $s \models \mathbf{AG}\Phi$ iff $\forall \pi \in \text{Path}(\mathcal{S}, s). \forall j. \pi(j) \models \Phi$
 - $s \models \mathbf{AF}\Phi$ iff $\forall \pi \in \text{Path}(\mathcal{S}, s). \exists j. \pi(j) \models \Phi$
 - analogously for **AU**, **AR**, **AW**
 - just replace $\forall$ with $\exists$ for **EF**, **ER**, **EW**
- Analogously to LTL, for many CTL formulas it is silently required that paths are infinite
- So again transition relations in Ks must be total



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Safety and Liveness Properties in CTL

- Some CTL formulas may be neither safety nor liveness
 - being defined on states, the counterexample may be an entire computation tree
- Safety properties are those involving only **AG**, **AX**, true and atomic propositions
- Some formulas are both safety and liveness, like true, **AG** true and so on
- Liveness are formulas like **AF**, **AFAG**, **AU**
- **EF** or **EG** are neither liveness nor safety

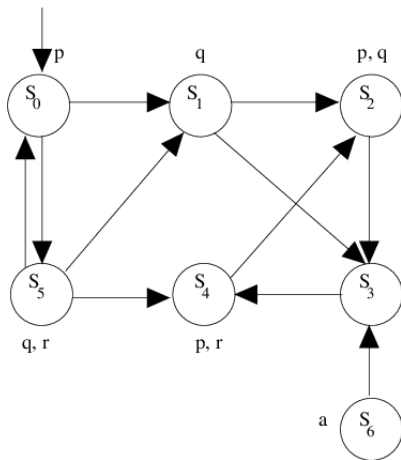


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CTL Examples



$\mathcal{S} \models \mathbf{AF}p$ since p holds in the first state

For full: $s_0 \models \mathbf{F}p$ since $p \in L(s_0)$, thus, for all paths starting in s_0 , p holds in the first state, so it holds eventually

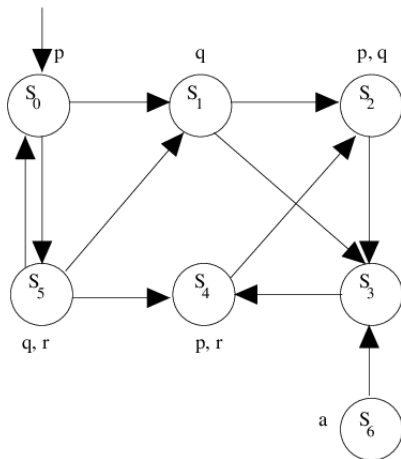


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CTL Examples



$\mathcal{S} \models \mathbf{EF}p$ for the same reason as above

If it holds for all paths, then it holds for one path

$\mathbf{AF}\phi \rightarrow \mathbf{EF}\phi$

The same holds for the other temporal operators **G**, **U** etc

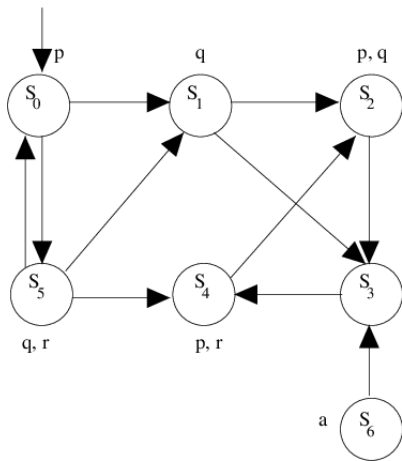


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CTL Examples



$\mathcal{S} \not\models \mathbf{EF}a$ since s_6 is not reachable

Note that the counterexample cannot be a single path

Since it would not enough to disprove existence

The full reachable graph must be provided

One could also show the tree of all paths

Neither safety nor liveness

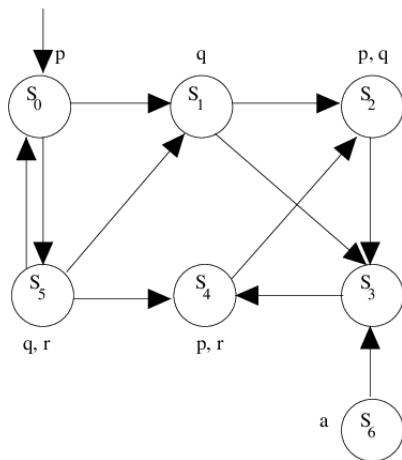


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CTL Examples



$\mathcal{S} \models \mathbf{A}(p \mathbf{U} q)$ since $p \in L(s_0)$,
 $\text{next}(s_0) = \{s_1, s_5\}$ and $q \in L(s_1) \wedge q \in L(s_5)$

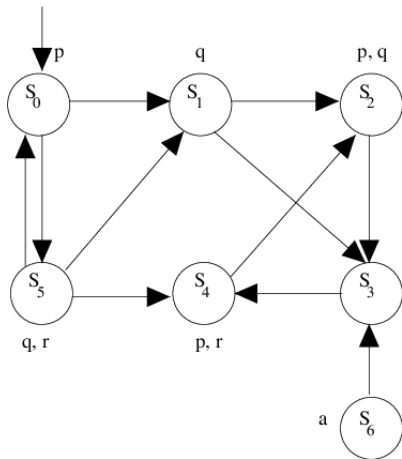


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CTL Examples



$\mathcal{S} \not\models \mathbf{A}(p \mathbf{U} r)$, a counterexample is $\pi = s_0 s_1 (s_2 s_3 s_4)$

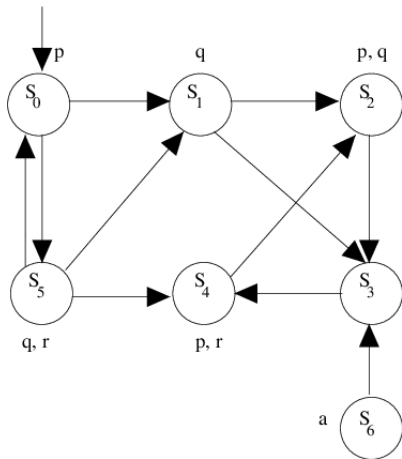


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CTL Examples



$\mathcal{S} \models \mathbf{E}(p \mathbf{U} r)$, an example is
 $\pi = (s_0 s_5)$

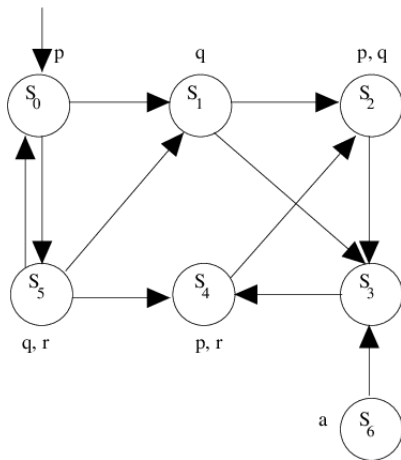


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CTL Examples



$\mathcal{S} \not\models \neg \mathbf{E}(p \mathbf{U} r)$, a counterexample is $\pi = (s_0 s_5)$

In fact, $\mathcal{S} \not\models \Phi$ iff $\mathcal{S} \models \neg(\Phi)$

Because here we have a single initial state

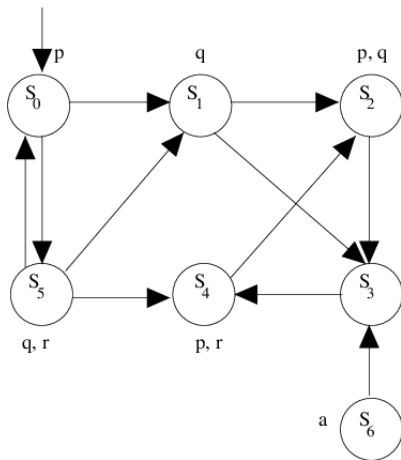


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CTL Examples



$\mathcal{S} \not\models \mathbf{AFAG}p$, a counterexample is $\pi = s_0 s_1 (s_2 s_3 s_4)$
This is a liveness formula

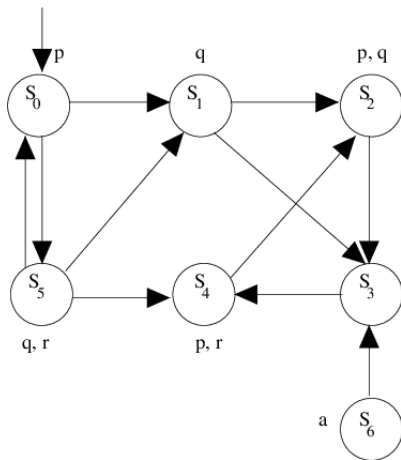


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CTL Examples



$\mathcal{S} \not\models \mathbf{EFEG}p$, a counterexample is again a computation tree

All lassos are s_0s_5 or $s_2s_3s_4$

In both such lassos, there are states in which p does not hold

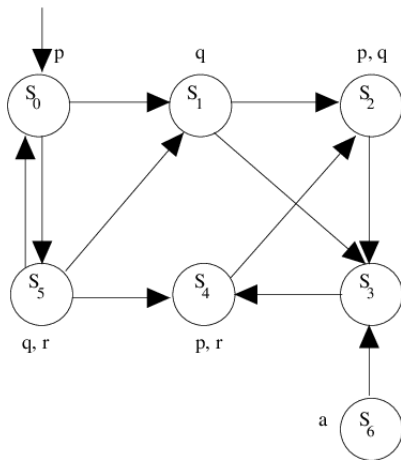


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CTL Examples



$\mathcal{S} \not\models \mathbf{AFEG}p$, a counterexample is again a computation tree
Since $\mathcal{S} \not\models \mathbf{EFEG}p \dots$

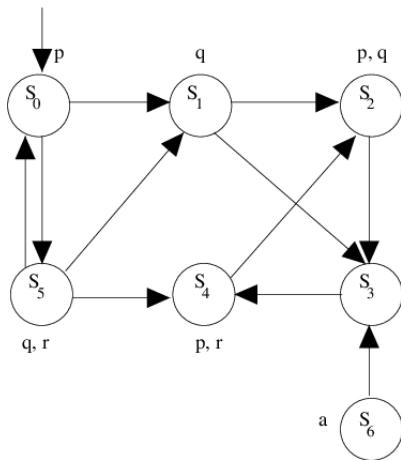


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CTL Examples



$\mathcal{S} \not\models \mathbf{EFAG}p$, a counterexample is again a computation tree
 Since $\mathcal{S} \not\models \mathbf{EFEG}p$...



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CTL Non-Toy Examples

- Recall the Peterson's protocol: checking mutual exclusion is **AG**($\neg(p \wedge q)$), being $p = P[1] = L3, q = P[2] = L3$
 - equivalent to LTL **G** p
- It is always possible to restart:
AGEF $P[1] = L0 \wedge \mathbf{AGEF} P[2] = L0$



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CTL vs. LTL: a Comparison

- Recall that $\varphi_1 \equiv \varphi_2$ iff $\forall \mathcal{S}. \mathcal{S} \models \varphi_1 \Leftrightarrow \mathcal{S} \models \varphi_2$
 - also holds (w.l.g.) when φ_1 is LTL and φ_2 is CTL
- Of course, some CTL formulas cannot be expressed in LTL
 - it is enough to put an **E**, since LTL always universally quantifies paths
 - so, there is not an LTL φ s.t. $\varphi \equiv \mathbf{EG}p$
 - no, $\mathbf{F}\neg p$ is not the same, why?
- So, one might think: LTL is contained in CTL
 - simply replace each temporal operator **O** with **AO**, that's it
 - let $\mathcal{T}$ be a translator doing this
 - for any LTL formula φ , $\varphi \equiv \mathcal{T}(\varphi)$
 - actually, $\mathbf{G}p \equiv \mathcal{T}(\mathbf{G}p) = \mathbf{AG}p$



CTL vs. LTL: a Comparison

- Theorem. Let φ be an LTL formula. Then, either i) $\varphi \equiv \mathcal{T}(\varphi)$ or ii) there does not exist a CTL formula ψ s.t. $\varphi \equiv \psi$
 - idea of proof: replacing with **E** is of course not correct, and temporal operators on paths are the same
- Corollary. There exists an LTL formula φ s.t., for all CTL formulas ψ , $\varphi \not\equiv \psi$
- Proof of corollary:
 - by the theorem above and the definitions, we need to find
 - 1 an LTL formula φ
 - 2 a KS $\mathcal{S}$
 - where $\mathcal{S} \models \varphi$ and $\mathcal{S} \not\models \mathcal{T}(\varphi)$
 - viceversa is not possible



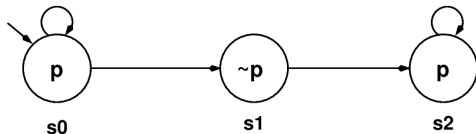
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CTL vs. LTL: a Comparison

- For example, as for the LTL formula, we may take $\varphi = \mathbf{FG}p$
 - note instead that $\mathbf{GF}p \equiv \mathbf{AGAF}p$
- For example, as for the KS $\mathcal{S}$, we may take



- We have that $\mathcal{S} \models \mathbf{FG}p$, but $\mathcal{S} \not\models \mathbf{AFAG}p$
- Thus, CTL requires “more” than the corresponding LTL

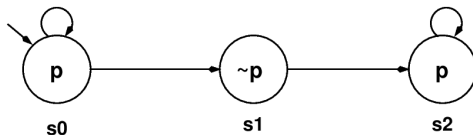


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CTL vs. LTL: a Comparison



- $\mathcal{S} \not\models \mathbf{AFAG}p$ means that
$$\neg(\forall \pi \in \text{Path}(\mathcal{S}). \exists j : \forall \rho \in \text{Path}(\mathcal{S}, \pi(j)). \forall k. p \in \rho(k))$$
$$= \exists \pi \in \text{Path}(\mathcal{S}). \forall j : \exists \rho \in \text{Path}(\mathcal{S}, \pi(j)). \exists k. p \notin \rho(k)$$
- In our $\mathcal{S}$, $\pi = s_0^\omega$: in fact, at any point of π , you may branch and go through $\neg p$ instead...
- $\mathcal{S} \models \mathbf{FG}p$ means that $\forall \pi \in \text{Path}(\mathcal{S}). \exists j : \forall k \geq j. p \in \pi(k)$
- Thus, there is not a CTL formula equivalent to $\mathbf{FG}p$
- Furthermore, there is not an LTL formula equivalent to $\mathbf{AFAG}p$

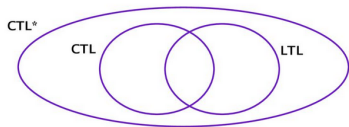


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CTL, LTL and CTL*



- CTL* introduced in 1986 (Emerson, Halpern) to include both CTL and LTL
- No restrictions on path quantifiers to be 1-1 with temporal operators, as in CTL
- State formulas: $\Phi ::= \text{true} \mid p \mid \Phi_1 \wedge \Phi_2 \mid \neg \Phi \mid \mathbf{A}\Psi \mid \mathbf{E}\Psi$
- Path formulas: $\Psi ::= \Phi \mid \Psi_1 \wedge \Psi_2 \mid \neg \Psi \mid \Psi_1 \mathbf{U} \Psi_2 \mid \mathbf{F}\Psi \mid \mathbf{G}\Psi$

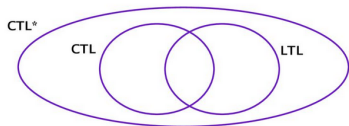


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CTL, LTL and CTL*



- The intersection between CTL and LTL is both syntactic and “semantic”
- Some formulas are both CTL and LTL in syntax: all those involving only boolean combinations of atomic propositions
- “Semantic” intersection: some LTL formulas may be expressed in CTL and vice versa, using different syntax
 - **AGAF** p and **GF** p
 - **AG** p and **G** p
 - etc



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Acronyms

- Murphi stands for nothing, though it is probable that it reminds Murphi's Laws
 - “if something may fail, it will fail”, i.e., $\mathbf{EF}p \rightarrow \mathbf{AF}p$
- SPIN stands for Simple Promela INTERpreter
- Promela is the SPIN input language
 - Murphi input language does not have a proper name
- Promela stands for PROcess MEta LANGUAGE
 - as we will see, it is actually based on Operating Systems-like processes
- Also see slides at
<https://spinroot.com/spin/Doc/SpinTutorial.pdf>
 - some as reused here



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Structure of a Promela Model

- We recall that Murphi input language is based on:
 - global variables with finite types
 - base types are integer subranges and enumerations
 - higher types are arrays and structures
 - function and procedures
 - guarded rules and starting states (*dynamics*)
 - may call functions and procedures, in an *atomic* way
 - Pascal-like syntax: `:=` for assignments, `=` for equality checks...
 - invariants



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Structure of a Promela Model

- Promela instead has:
 - global variables with finite types
 - base types are integer types of the C language
 - enumerations are very limited
 - arrays and records
 - channels!
 - processes behaviour (*dynamics*)
 - possibly with arguments and local variables
 - properties to be checked:
 - assertions
 - deadlocks
 - “neverclaim” describing a BA
 - a separate tool may translate an LTL formula in the corresponding BA



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Peterson Protocol in Promela

```
bool turn, flag[2];
byte ncrit;

active [2] proctype user()
{
  assert(_pid == 0 || _pid == 1);
again:
  flag[_pid] = 1;
  turn = _pid;
  (flag[1 - _pid] == 0 || turn == 1 - _pid);
  ncrit++;
  assert(ncrit == 1); /* critical section */
  ncrit--;
  flag[_pid] = 0;
  goto again
}
```



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Dijkstra Protocol in Promela

```
#define p 0
#define v 1
chan sema = [0] of { bit }; /* rendez-vous */

proctype dijkstra()
{
    byte count = 1; /* local variable */
    do
        :: (count == 1) -> sema!p; count = 0
        /* send 0 and blocks, unless some other
           proc is already blocked in reception */
        :: (count == 0) -> sema?v; count = 1
        /* receive 1, same as above */
    od
}
```



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Dijkstra Protocol in Promela

```
proctype user()  
{  
  do  
    :: sema?p;  
      /*      critical section      */  
      sema!v;  
      /* non-critical section */  
  od  
}  
  
init  
{  
  run dijkstra();  
  run user(); run user(); run user()  
}
```



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SPIN Simulation

Almost equal to Murphi one

```
void Make_a_run(NFSS  $\mathcal{N}$ )
{
  let  $\mathcal{N} = \langle S, \{s_0\}, \text{Post} \rangle$ ;
  s_curr =  $s_0$ ;
  if (some assertion fail in s_curr)
    return with error message;
  while (1) { /* loop forever */
    if (Post(s_curr) =  $\emptyset$ )
      return with deadlock message;
    s_next = pick_a_state(Post(s_curr));
    if (some assertion fail in s_curr)
      return with error message;
    s_curr = s_next;
  }
}
```



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SPIN Verification

- Able to answer to the following questions:
 - is there a deadlock (invalid end state)?
 - are there reachable assertions which fail (safety)?
 - is a given LTL formula (safety or liveness) ok in the current system?
 - is a given neverclaim (safety or liveness) ok in the current system?
- It is possible to specify some side behaviours:
 - is sending to a full channel blocking, or the message is dropped without blocking?
- It may report unreachable code
 - Promela statements in the model which are never executed



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SPIN Verification

- Similar to Murphi:
 - 1 the SPIN compiler (`SrcXXX/spin -a`) is invoked on `model.prm` and outputs 5 files:
 - `pan.c`, `pan.h`, `pan.m`, `pan.b`, `pan.t` (unless there are errors...)
 - 2 the 5 files given above are compiled with a C compiler
 - it is sufficient to compile `pan.c`, which includes all other files
 - in this way, an executable file `model` is obtained
 - 3 just execute `model`
 - option `--help` gives an overview of all possible options



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SPIN Verification of LTL Formulas

- The former is ok for assertion or deadlock checks
- If you also have an LTL formula
 - 1 the SPIN compiler (`SrcXXX/spin -F`) is invoked on `model.ltl` and outputs a neverclaim on the standard output
 - `model.ltl` must be a text file with only 1 line
 - file extensions does not matter
 - syntax for the formula: **G** is `[]`, **F** is `<>`, **U** is `U`
 - atomic propositions must be identifiers
 - 2 append the neverclaim to the promela file
 - 3 define the identifiers used as atomic proposition by `#defines` in the promela file
 - 4 go on as before



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Standard Recursive DFS

```
HashTable Visited =  $\emptyset$ ;
```

```
DFS(graph  $G = (V, E)$ , node  $v$ )  
{  
    Visited := Visited  $\cup v$ ;  
    foreach  $v' \in V$  t.c.  $(v, v') \in E$  {  
        if ( $v' \notin$  Visited)  
            DFS( $G, v'$ );  
    }  
}
```



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Iterative DFS Easy Version

```
DFS(graph  $G = (V, E)$ )
{
    s := init;
    push(s, 1);
    while (stack  $\neq \emptyset$ ) {
        (s, i) := top();
        increment i on the top of the stack;
        if (s  $\notin$  Visited) {
            Visited := Visited  $\cup$  s;
            let  $S' = \{s' \mid (s, s') \in E\}$ ;
            if ( $|S'| \geq i$ ) {
                s := i-th element in  $S'$ ;
                push(s, 1);
            }
            else pop();
        }
        else pop();
    }
    else pop();
} }
```



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Iterative DFS

```
DFS(graph  $G = (V, E)$ )
{
     $s := \text{init}$ ;  $i := 1$ ;  $\text{depth} := 0$ ;
    push( $s$ , 1);
Down:
    if ( $s \in \text{Visited}$ )
        goto Up;
    Visited := Visited  $\cup s$ ;
    let  $S' = \{s' \mid (s, s') \in E\}$ ;
    if ( $|S'| \geq i$ ) {
         $s := i\text{-th element in } S'$ ;
        increment  $i$  on the top of the stack;
        push( $s$ , 1);
         $\text{depth} := \text{depth} + 1$ ;
        goto Down;
    }
}
```



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Iterative DFS

```
Up:
  (s, i) := pop();
  depth := depth - 1;
  if (depth > 0)
    goto Down;
}
```



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Partial Order Reduction

- POR does not try to use less memory to save the same states: it tries to save less states
 - while retaining correctness, of course
 - some states are “useless” and need not to be explored (and saved)
 - also saves in computation time, of course
- Similar to Murphi symmetry for the goal, but different in use and algorithm
 - use: Murphi modeler must specify which parts of the model are symmetric
 - in SPIN, POR is directly applied without the modeler being aware of it
 - though it is possible to disable it



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CTL (and LTL) Model Checking

- We saw the theoretical algorithm for CTL model checking
 - we said it was not effective, as it required S and R to be in RAM
- Actually, there are methodologies which are able to fit S and R in RAM, also for industrial-sized models
- The “father” of the model checkers using such technologies is SMV
 - Symbolic Model Verifier
 - it has then been refactored as NuSMV
- This set of techniques is referred to as *symbolic model checking*
 - Murphi and SPIN style is dubbed *explicit model checking*



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CTL (and LTL) Model Checking

- In order to understand how symbolic model checking works, we need some preliminaries
- ROBDDs
 - needed to actually fit S and R in RAM
- μ -calculus
 - together with fixpoint computation
 - extension of λ -calculus
 - needed to efficiently implement CTL and LTL model checking using ROBDDs



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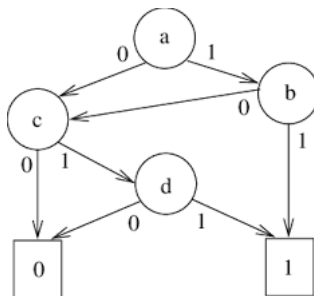


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- Reduced Ordered (Complemented Edges) Binary Decision Diagrams
 - sometimes called simply OBDDs, and even BDDs
 - here we stick to the precise notation, by also outlining the differences
- Let us start with the basis: BDD
- A BDD is a data structure representing a boolean function
 - of course, OBDDs and ROBDDs are data structures as well
 - we will define them in the following



Binary Decision Diagrams



Represented function: $f(a, b, c, d) = ab + \bar{a}cd + a\bar{b}cd$

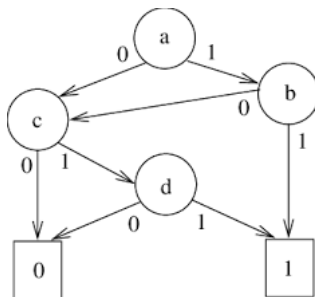


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OBDDs



Supposing that $V = \mathcal{V}$, a possible ordering is:

$\text{ord}(a) = 1, \text{ord}(b) = 2, \text{ord}(c) = 3, \text{ord}(d) = 4$

If b were connected to d instead of c , also:

$\text{ord}(a) = 1, \text{ord}(b) = 3, \text{ord}(c) = 2, \text{ord}(d) = 4$

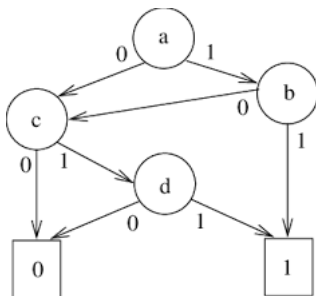


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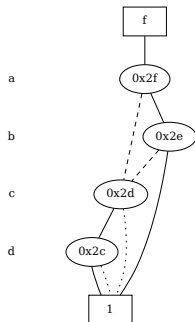


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COBDDs



Represented function:
 $f(a, b, c, d) = ab + \bar{a}cd + \bar{a}\bar{b}cd$



straight: then, dashed: else,
 dotted: complemented else



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NuSMV Input Language

Taken from `examples/smv-dist/short.smv`

```
MODULE main
```

```
VAR
```

```
    request : {Tr, Fa}; -- same as saying boolean
                        -- (stand for True and False)
```

```
    state : {ready, busy};
```

```
ASSIGN
```

```
    init(state) := ready;
```

```
    next(state) := case
```

```
        state = ready & (request = Tr): busy;
```

```
        TRUE : {ready, busy};
```

```
    esac;
```

```
SPEC
```

```
    AG((request = Tr) -> AF state = busy)
```

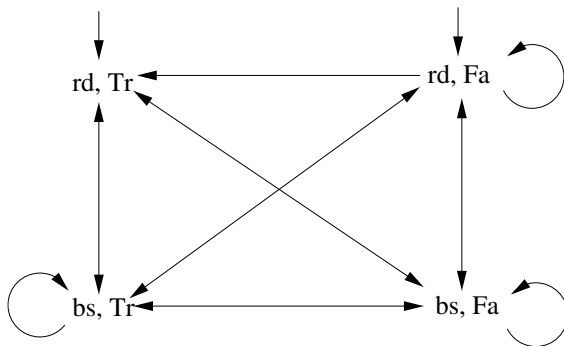


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Automata for short.smv: I and R



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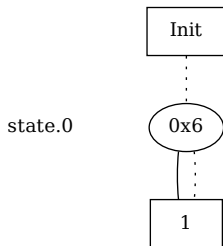
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OBDDs for short.smv: /

Straight lines are then-edges

Dashed lines are else-edges

Dotted lines are complemented-else-edges



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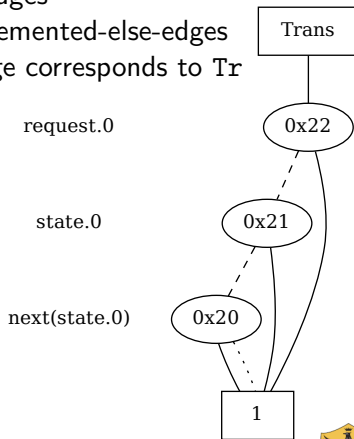
OBDDs for short.smv: R

Straight lines are then-edges

Dashed lines are else-edges

Dotted lines are complemented-else-edges

request.0 “false” edge corresponds to Tr



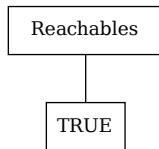
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OBDDs for short.smv: Reach

The one for soloready is the same

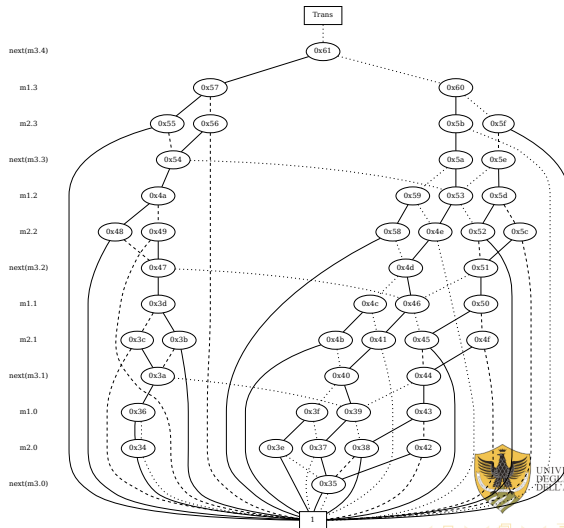


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OBDDs for Adder: R

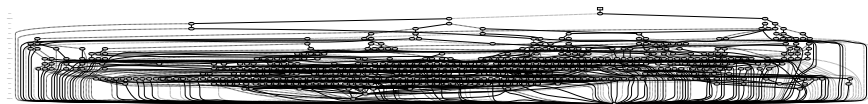


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OBDDs for Multiplier: R



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OBDDs Pros and Cons

- Number of variables is 13 for both models
 - 4 each for m_1 and m_2 , plus 5 for m_3
- Number of BDD nodes:
 - adder: 47
 - multiplier: 538



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OBDDs Pros and Cons

- No magic: SAT could be solved using OBDDs
 - just represent the instance with an OBDD and check if it is different from 0
 - very roughly speaking: if it were possible to solve it “efficiently” in this way, $P=NP$...
- Thus, there are boolean functions for which OBDDs representation is exponential, regardless of variable ordering
 - one example is the multiplier seen above
- It is not possible to say if OBDDs will be a good way to represent a problem, before trying it
 - for the adder, it is much more efficient
- Furthermore, finding a variable order in order to minimize the OBDD representation for a given function is an NP-complete problem



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OBDDs Pros and Cons

- This also holds for Model Checking in general
- Not possible to say a-priori if a system will fit in the available resources when using a model checker
 - RAM and computation time
- Also, it is not possible to decide which model checker is better
 - explicit (Murphi-or-SPIN like) or symbolic (NuSMV like)?
- However, we are going to see some guidelines
 - as for OBDDs: a good ordering is to interleave present and future variables
 - variable ordering: if OBDDs grow, the model checker can try a different variable ordering



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Computation of Least (Minimum) Fixpoint

```
OBDD lfp(MuFormula T) /*  $\mu Z.T(Z)$  */  
{  
  Q =  $\lambda x.0$ ;  
  Q' = T(Q);  
  /* T clearly says where Q must be replaced */  
  /* e.g.: if  $\mu Z.\lambda x.f(x) \vee Z(x)$ , then  
    Q' =  $\lambda x.f(x) \vee Q(x)$  */  
  while (Q  $\neq$  Q') {  
    Q = Q';  
    Q' = T(Q);  
  }  
  return Q; /* or Q', they are the same... */  
}
```



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Computation of Greatest (Maximum) Fixpoint

```
OBDD gfp(NuFormula T) /*  $\nu Z.T(Z)$  */  
{  
  Q =  $\lambda x.1$ ;  
  Q' = T(Q);  
  while (Q  $\neq$  Q') {  
    Q = Q';  
    Q' = T(Q);  
  }  
  return Q;  
}
```



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Symbolic Model Checking of $\mathbf{AG}p$

- The idea is to compute the set of reachable states, and check if for all of them p holds
- $\text{Reach} = \mu Z. \lambda x. [I(x) \vee \exists y : (Z(y) \wedge R(y, x))]$
 - of course, we get an OBDD on x as a result
 - recall that x (and y) is a vector of all boolean variables
- $\forall x \in S. \text{Reach}(x) \rightarrow p(x)$
 - computationally easier: check that $\text{Reach}(x) \wedge \neg p(x) = 0$
 - otherwise, we have a reachable state for which p does not hold...



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CTL Model Checking

```
bool checkCTL(KS S, CTL  $\varphi$ ) {  
  let  $S = \langle S, I, R, L \rangle$ ;  
   $B = \text{Lb1St}(\varphi)$ ;  
  return  $\lambda x. I(x) \wedge \neg B(x) = \lambda x. 0$ ;  
}  
  
OBDD Lb1St(CTL  $\varphi$ ) { /* also  $S = \langle S, I, R, L \rangle$  */  
  if ( $\exists p \in AP. \varphi = p$ ) return  $\lambda x. p(x)$ ;  
  else if ( $\varphi = \neg\phi$ ) return  $\lambda x. \neg \text{Lb1St}(\phi)(x)$ ;  
  else if ( $\varphi = \phi_1 \wedge \phi_2$ )  
    return  $\lambda x. \text{Lb1St}(\phi_1)(x) \wedge \text{Lb1St}(\phi_2)(x)$ ;  
  else if ( $\varphi = \mathbf{EX}\phi$ )  
    return  $\lambda x. \exists y : R(x, y) \wedge \text{Lb1St}(\phi)(y)$ ;  
  else if ( $\varphi = \mathbf{EG}\phi$ )  
    return  $\text{gfp}(\nu Z. \lambda x. \text{Lb1St}(\phi)(x) \wedge (\exists y : R(x, y) \wedge Z(y)))$ ;  
  else if ( $\varphi = \phi_1 \mathbf{EU} \phi_2$ )  
    return  $\text{lfp}(\mu Z. \lambda x. \text{Lb1St}(\phi_2)(x) \vee$   
       $(\text{Lb1St}(\phi_1)(x) \wedge (\exists y : R(x, y) \wedge Z(y))))$ ;  
}
```



Towards Bounded Model Checking

- Explicit and symbolic model checking are good, but many systems cannot be checked by neither
 - RAM and/or execution time are over soon
- Symbolic model checking directly makes use of boolean formulas through OBDDs
- What about using CNF, so that SAT solvers can be employed?
 - modern SAT solvers are pretty good in many practical instances
 - notwithstanding the SAT problem is of course still NP-complete



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Towards Bounded Model Checking

- One big problem: computing quantization, AND, OR and negation of a CNF is not straightforward
 - especially because instances from Model Checking are HUGE
 - also checking equivalence of two CNF is not trivial, as CNF is not canonical
- However, if we set a limit k to the length of paths (counterexamples), then it is easy
 - copy R for k times, with small adjustments
- This is actually *bug hunting*: if the result is PASS, then there is not an error within k steps
 - but there could be one at $k + 1...$
 - however, this is better than simple testing, as errors within k steps can be ruled out



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Bounded Model Checking of Safety Properties

- In Bounded Model Checking (BMC) we are given a KS $\mathcal{S} = \langle S, I, R, L \rangle$, an LTL formula φ , and $k \in \mathbb{N}$ (also called *horizon*)
- Let us consider the LTL property $\varphi = \mathbf{G}p$, being $p \in AP$
- We want to find counterexamples (if any) of length exactly k
- If $x = x_1, \dots, x_n$ with $n = \lceil \log_2 |S| \rceil$, let us consider $x^{(0)}, \dots, x^{(k)}$
- $\mathcal{S} \models_k \mathbf{G}p$ iff the following CNF is unsatisfiable:

$$I(x^{(0)}) \wedge \bigwedge_{i=0}^{k-1} R(x^{(i)}, x^{(i+1)}) \wedge \neg p(x^{(k)})$$

- otherwise, a satisfying assignment is a counterexample



Bounded Model Checking of Safety Properties

- Note that each $x^{(i)}$ encloses n boolean variables, thus we have $n(k + 1)$ boolean variables in our SAT instance
 - the longest our horizon, the biggest our SAT instance
- Note that I and R must be in CNF, which is not difficult
 - NuSMV does this pretty well
- It is straightforward to modify the previous formula to detect counterexamples of length *at most* k
- However, it is usually preferred to perform BMC with increasing values for k
 - practically, till when the SAT solver goes out of computational resources
 - some approaches exist to estimate the *diameter* of a KS...



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Bounded Model Checking of Programs

- Till now, we had to write a model of the system under verification (SUV)
- There are some cases in which we can use the actual SUV, with little or no instrumentation
 - it is possible to translate a digital circuit to a NuSMV specification in a completely automated way (not difficult to imagine how...)
 - here, we want to deal with a rather surprising application of BMC: model checking a C program!
- CBMC is a model checker performing BMC of C programs with little or no instrumentation
 - thus, the input for CBMC is a C program (possibly with some added statements)
 - an integer k may be required too
 - again, output is PASS or FAIL (with a counterexample)
- We now give the main ideas of how it works

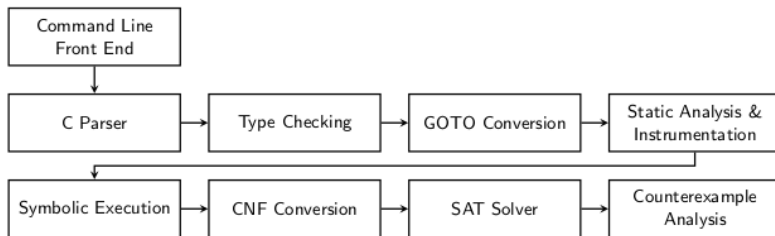


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CBMC



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