Software Testing and Validation

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Corso di Laurea in Informatica

The NuSMV Model Checker

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CTL (and LTL) Model Checking

- We saw the theoretical algorithm for CTL model checking
    - we said it was not effective, as it required S and R to be in RAM
- Actually, there are methodologies which are able to fit S and R in RAM, also for industrial-sized models
- The “father” of the model checkers using such technologies is SMV
    - Symbolic Model Verifier
    - it has then been refactored as NuSMV
- This set of techniques is referred to as *symbolic model checking*
    - Murphi and SPIN style is dubbed *explicit model checking*

CTL (and LTL) Model Checking

- In order to understand how symbolic model checking works, we need some preliminaries
- ROBDDs
    - needed to actually fit S and R in RAM
- μ-calculus
    - together with fixpoint computation
    - extension of λ-calculus
    - needed to efficiently implement CTL and LTL model checking using ROBDDs

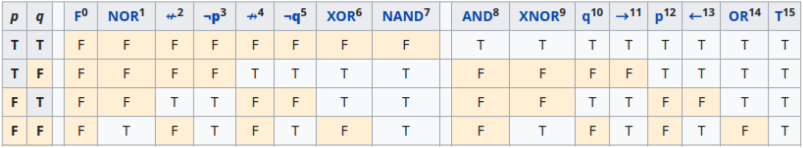
ROBDD

- Reduced Ordered (Complemented Edges) Binary Decision Diagrams
  - sometimes called simply OBDDs, and even BDDs
  - here we stick to the precise notation, by also outlining the differences
- Let us start with the basis: BDD
- A BDD is a data structure representing a boolean function
  - of course, OBDDs and ROBDDs are data structures as well
  - we will define them in the following

Boolean Functions

- In our setting a boolean function is $f : \mathbb{B}^n \rightarrow \mathbb{B}$
  - where $\mathbb{B} = \{0, 1\}$ is the set of boolean values
  - 0 stands for false, 1 for true
  - thus, our boolean functions have n boolean variables as arguments
  - and return a single boolean value
- Examples:
  - 0 and 1 are boolean functions with $n = 0$
  - complementation ($f(x) = \neg x$) and identity ($f(x) = x$) are boolean functions with $n = 1$
  - AND ($f(x, y) = x \wedge y$), OR ($f(x, y) = x \vee y$) are boolean functions with $n = 2$
  - generally speaking, there are 2^{2^n} different boolean functions of n boolean variables

All Boolean Functions of 2 Variables



p	*q*		F^{0}	NOR^{1}	$\nleftarrow^{2}$	$\neg p^{3}$	$\nrightarrow^{4}$	$\neg q^{5}$	XOR^{6}	NAND^{7}		AND^{8}	XNOR^{9}	q^{10}	$\rightarrow^{11}$	p^{12}	$\leftarrow^{13}$	OR^{14}	T^{15}
T	**T**		F	F	F	F	F	F	F	F		T	T	T	T	T	T	T	T
T	**F**		F	F	F	F	T	T	T	T		F	F	F	F	T	T	T	T
F	**T**		F	F	T	T	F	F	T	T		F	F	T	T	F	F	T	T
F	**F**		F	T	F	T	F	T	F	T		F	T	F	T	F	T	F	T

Boolean Functions Representation

- Roughly speaking, if you have $f(x) = x + 1$ with $x \in \mathbb{R}$, you can only represent f through its computation
  - rules s.t., given x, you compute $x + 1$
- For boolean functions, the explicit tabular representation is also possible (*truth table*)
  - a table with $n + 1$ columns
  - first n columns are for variables values
  - last column is for function value
  - of course, you need 2^n rows
  - actually, only one column, thus $\lceil 2^{n-3} \rceil$ bytes
    - thus, $O(2^n)$

Boolean Functions Representation

- A truth table must take into account all possible values for all its n arguments
- Which leads to a $O(2^n)$ RAM required
  - even with optimizations (e.g., only 1 column is actually needed)
- This also implies $O(2^n)$ time to compute composition of functions
  - e.g., $f \wedge g$
  - worst case time is also best case...
- One very good thing about truth tables: they are *canonical*
  - for a function f, given the (standard) order of the lines, there is only one truth table

Boolean Functions Representation

- What about CNF or DNF?
  - CNF: $(x_1 + x_2)(\bar{x}_3 + x_4)$
  - DNF: $x_1\bar{x}_3 + x_1x_4 + x_2\bar{x}_3 + x_2x_4$
  - recall that $+$ is OR, $\cdot$ is AND, $\bar{\cdot}$ is negation
- Approx ok to compute function compositions
- Difficult to obtain a minimal representation
- Above all, not canonical: there may be multiple CNFs or DNFs for the same function
  - also if you consider the minimal one

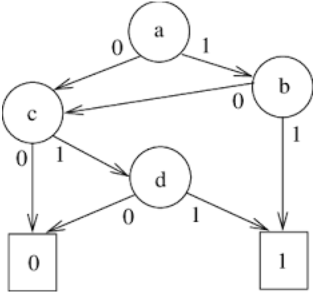
Boolean Functions for Model Checking

- In Model Checking algorithms, the following operations are needed:
  - compute the returned value for a given tuple of values $b_1, \ldots, b_n$
    - could be ok for truth tables and DNF/CNF
  - test of equivalence between boolean functions $f_1 \equiv f_2$
    - not ok neither for truth tables nor for CNF/DNF
  - compute the representation of a logical combination of boolean functions
    - e.g.: given the representation of f_1, f_2, compute the representation of $f_1 \wedge f_2$
    - not ok for truth tables
    - slightly better for CNF/DNF
- Goal: find a representation able to fulfill such requirements
  - while possibly requiring less than $O(2^n)$ memory

Binary Decision Diagrams

- Roughly speaking, it is a connected DAG (Directed Acyclic Graph), i.e., a tree
    - only one root
    - each internal node has two successors
    - nodes are labeled by boolean variables
    - edges are labeled by boolean values
    - only two leaves, labeled with boolean values

Binary Decision Diagrams


a
0
1
b
0
c
1
0
1
d
0
1
0
1

Represented function: $f(a,b,c,d) = ab + \bar{a}cd + a\bar{b}cd$

BDDs: Formal Definition

- A BDD is a tuple $\mathcal{B} = \langle V, E, r, \mathcal{V}, \mathrm{var}, \mathrm{low}, \mathrm{high} \rangle$ where:
  - V is a finite set of nodes containing two special nodes $\mathbf{0}$ and $\mathbf{1}$
  - $E \subseteq V \times V$ is a set of edges s.t.:
    - there are no cycles, i.e., for all path $\pi = v_0, \ldots, v_n$, where $\forall i = 0, \ldots, n.\ v_i \in V$ and $\forall i = 0, \ldots, n-1.\ (v_i, v_{i+1}) \in E$, we have that $i \neq j$ implies $v_i \neq v_j$
    - let $S(v) = \{w \in V \mid (v, w) \in E\}$ be the set of successors of v
    - each internal node has exactly two successors, i.e., $\forall v \in V \setminus \{\mathbf{0}, \mathbf{1}\}.\ |S(v)| = 2$
    - $\mathbf{0}$ and $\mathbf{1}$ are *terminal nodes*, i.e., $\forall v \in \{\mathbf{0}, \mathbf{1}\}.\ |S(v)| = 0$
  - $r \in V$ is the root (i.e., $\forall v \in V.\ (v, r) \notin E$)
  - $\mathrm{low}, \mathrm{high} : V \to V$ is the labeling of edges
    - the labeling must be consistent with E, i.e., $\forall v \in V.\ \mathrm{low}(v), \mathrm{high}(v) \in S(v)$

BDDs: Formal Definition

- A BDD is a tuple $\mathcal{B} = \langle V, E, r, \mathcal{V}, \mathrm{var}, \mathrm{low}, \mathrm{high} \rangle$ where:
  - $\mathcal{V}$ is a finite set of boolean variables
    - thus, the boolean function represented by $\mathcal{B}$ will depend on variables in $\mathcal{V}$
    - it may be a subset of $\mathcal{V}$
  - $\mathrm{var} : V \to \mathcal{V}$ is the labeling of nodes
- A *maximal path* in $\mathcal{B}$ starts from r and ends up either in **0** or **1**
- The *semantics* of $\mathcal{B}$ is the boolean function represented by $\mathcal{B}$
  - intuitively, we follow all maximal paths which end up in **1**
  - formally: next slide

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BDDs: Semantics

- Given a BDD $\mathcal{B} = \langle V, E, r, \mathcal{V}, \mathrm{var}, \mathrm{low}, \mathrm{high} \rangle$, we recursively define the semantics of each node $v \in V$
  - each node may be seen as the root of a subtree...
  - notation: $[\![v]\!]_{\mathcal{B}}$, or simply $[\![v]\!]$ when $\mathcal{B}$ is understood
- Terminal nodes denote the boolean constants: $[\![\mathbf{0}]\!] = \mathrm{false}, [\![\mathbf{1}]\!] = \mathrm{true}$
- For internal nodes $v \in V \setminus \{\mathbf{0}, \mathbf{1}\}$, semantics is defined as $[\![v]\!] = \mathrm{var}(v)[\![\mathrm{high}(v)]\!] + \overline{\mathrm{var}(v)}[\![\mathrm{low}(v)]\!]$
  - recall that $+$ is OR, $\cdot$ is AND, $\bar{\cdot}$ is negation
- The semantics of $\mathcal{B}$ is of course $[\![r]\!]$

Canonicity of BDDs

- For a given BDD $\mathcal{B}$, we have a unique represented boolean function
- Given a boolean function f, there is a BDD $\mathcal{B}$ representing f, i.e., $[\![r]\!]_{\mathcal{B}} = f$
- However, there may be a BDD $\mathcal{B}' \neq \mathcal{B}$ s.t. $[\![r']\!]_{\mathcal{B}'} = f$ as well
  - thus, BDDs are not canonical
- Thus, ROBDDs are introduced: by setting limitations, they achieve canonicity
  - for a boolean function f, there exists a *unique* ROBDD representing f
- Furthermore, for increasing efficiency, complemented edges are introduced
  - number of nodes is reduced

OBDDs

- An OBDD (Ordered BDD) $\mathcal{B} = \langle V, E, r, \mathcal{V}, \mathrm{var}, \mathrm{low}, \mathrm{high}, \mathrm{ord} \rangle$, is a BDD with an additional ord function
- Namely, $\mathrm{ord} : \mathcal{V} \to \{1, \ldots, |\mathcal{V}|\}$
- The following properties must hold
  - ord is injective, i.e., $\forall v, w \in \mathcal{V}.\ \mathrm{ord}(v) = \mathrm{ord}(w) \to v = w$
  - note that this implies that ord is indeed bijective...
  - defines an *ordering* on variables in $\mathcal{V}$, e.g., if $\mathrm{ord}(v) = 10$ then v is the tenth variable
  - given a path π on $\mathcal{B}$, variables on nodes follow ord
  - i.e., $\forall \pi = v_0, \ldots, v_n$ s.t. $\forall i = 0, \ldots, n.\ v_i \in V$ and $\forall i = 0, \ldots, n-1.\ (v_i, v_{i+1}) \in E$ and $v_n \notin \{\mathbf{0}, \mathbf{1}\}$, we have that $i < j$ implies $\mathrm{ord}(\mathrm{var}(v_i)) < \mathrm{ord}(\mathrm{var}(v_j))$

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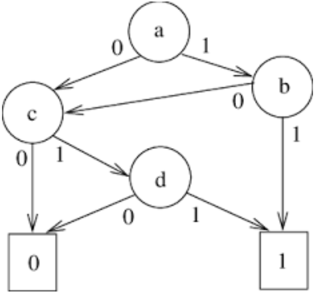
OBDDs



Supposing that $V = \mathcal{V}$, a possible ordering is:
$\mathrm{ord}(a) = 1, \mathrm{ord}(b) = 2, \mathrm{ord}(c) = 3, \mathrm{ord}(d) = 4$
If b were connected to d instead of c, also:
$\mathrm{ord}(a) = 1, \mathrm{ord}(b) = 3, \mathrm{ord}(c) = 2, \mathrm{ord}(d) = 4$

COBDDs

- A COBDD (Complemented edges OBDD) $\mathcal{B} = \langle V, E, r, \mathcal{V}, \mathrm{var}, \mathrm{low}, \mathrm{high}, \mathrm{ord}, \mathrm{flip} \rangle$, is an OBDD with an additional $\mathrm{flip} : V \setminus \{\mathbf{1}\} \to \{0, 1\}$
- For an internal node v, if $\mathrm{flip}(v)$ holds then the *else edge* of v is complemented
- There is now only one terminal node $\mathbf{1}$
  - $\mathbf{0}$ is not needed because of complementation
- Semantics changes, also a flipping bit $b \in \{0, 1\}$ is necessary
- Terminal node denote the boolean constants: $[\![\mathbf{1}, b]\!] = \bar{b}$
- For internal nodes $v \in V \setminus \{\mathbf{1}\}$, semantics is defined as $[\![v, b]\!] = \mathrm{var}(v)[\![\mathrm{high}(v), b]\!] + \overline{\mathrm{var}(v)}[\![\mathrm{low}(v), b \oplus \mathrm{flip}(v)]\!]$
- Semantics of $\mathcal{B}$ is $[\![r, \mathrm{flip}(r)]\!]$

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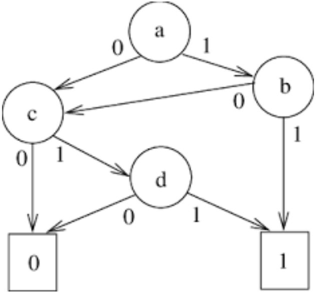
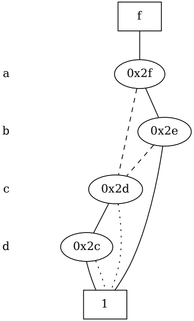
COBDDs


a
0
1
b
c
0
0
1
1
d
0
1
0
1

Represented function: $f(a, b, c, d) = ab + \bar{a}cd + a\bar{b}cd$


f
a
0x2f
b
0x2e
c
0x2d
d
0x2c
1

straight: then, dashed: else, dotted: complemented else

ROBDDs

- A ROBDD (Reduced OBDD) $\mathcal{B}$ is a COBDD with the least number of nodes
  - among the ones representing the same boolean function
- From now on, as usual in the literature, we will use OBDD as synonym for ROBDD
- Efficient algorithms ($O(n)$, being n the number of nodes) exist to compute the AND and the OR of two OBDDs
  - negation is $O(1)$: just complement $\mathrm{flip}(r)$!
- Typically implemented with hash tables of already computed ROBDDs
  - speedup computations, make it easier to find shared subtrees
  - equality check is $O(1)$: just compare r and r'
- Furthermore: multi-rooted DAG can be used to represent multiple functions, sharing some nodes

Other Important OBDD Operations

- Application: given the OBDD for $f(x_1, \dots, x_i, \dots, x_n)$, compute the OBDD for $f(x_1, \dots, 0, \dots, x_n)$ or $f(x_1, \dots, 1, \dots, x_n)$
    - sometimes also written $f(x_1, \dots, x_n)|_{x_i=0}$ or $f(x_1, \dots, x_n)|_{x_i=1}$
    - Shannon expansion: for every boolean function f, $f(x_1, \dots, x_n) = \bar{x}_i f(x_1, \dots, x_n)|_{x_i=0} + x_i f(x_1, \dots, x_n)|_{x_i=1}$
- Given $f(x, y)$, compute the OBDD for:
    - existentialization: $\exists x : \; f(x, y) \equiv f(0, y) + f(1, y)$
    - universalization: $\forall x. \; f(x, y) \equiv f(0, y) \cdot f(1, y)$
    - both generalized to multiple variables $x_1, \dots, x_n$
- Given $f(x), g(x), h(x)$, compute the OBDD for $ITE(f, g, h)$
    - ITE stands for if-then-else
    - thus, $ITE(f, g, h) = fg + \bar{f}h$

OBDD and Model Checking

- OBDDs extremely good in representing *characteristic functions* of finite sets
    - the characteristic function $\chi : U \to \{0, 1\}$ of a set $X \subseteq U$ is defined as

$$\chi(x) = \begin{cases} 1 & \text{if } x \in X \\ 0 & \text{otherwise} \end{cases}$$

- If U is finite, then each element $x \in U$ may be encoded using $n = \lceil \log(|U|) \rceil$ boolean variables $x_1, \ldots, x_n$
- Thus, χ may be represented by an OBDD on $x_1, \ldots, x_n$
    - as for Model Checking, we may represent S, $\mathrm{Reach}(\mathcal{S})$, R, ...
    - R will need $2n$ variables!
    - CTL Model Checking algorithm becomes feasible!
        - for many interesting real-sized systems, S, $\mathrm{Reach}(\mathcal{S})$, R will now fit in RAM

OBDD and Model Checking

- The most difficult part is to derive the OBDD for R directly from the model specification
  - i.e., from the model checker input language
  - it would be rather difficult to do it with SPIN
    - especially because it has a dynamic state space
  - also the one for Murphi would require some effort
  - S is easy, you only have to look at global variables
    - not in SPIN...
- NuSMV input language is tailored to be easily translated into OBDDs
  - also into CNF, as we will see...

NuSMV

- SMV (Symbolic Model Verifier): McMillan implementation of the ideas in the famous paper "Symbolic model checking: 10^{20} states and beyond"
  - McMillan PhD dissertation about SMV is one of the most important dissertations in Computer Science
- SMV has been then re-written and standardized by the research group in Trento (also Genova and CMU collaborated), thus becoming NuSMV
  - the engine is still McMillan's work
  - code has been nearly entirely commented, and made more readable
  - some features has been added: interactive mode, bounded model checking
  - OBDDs are handled via the CUDD library (by F. Somenzi at Colorado University)

NuSMV Input Language

Taken from `examples/smv-dist/short.smv`

```
MODULE main
VAR
  request : {Tr, Fa}; -- same as saying boolean
                      -- (stand for True and False)
  state : {ready, busy};
ASSIGN
  init(state) := ready;
  next(state) := case
                   state = ready & (request = Tr): busy;
                   TRUE : {ready,busy};
                 esac;
SPEC
  AG((request = Tr) -> AF state = busy)
```

NuSMV Input Language

- One *module*, there may be more, but one of them must be named `main`
- Module variables are those declared with `VAR`
- Base types are like Murphi ones: enumerations and integer subranges, plus the `word` type (i.e., an array of bits)
- Arrays are possible, but can be indexed only with constants
- Structures are modeled through modules
  - that is, each module has its variables (fields of a structure) and may be instantiated many times

NuSMV Input Language

- `ASSIGN` section specifies the set I (via `init`) and the relation R (via `next`)
  - as in Murphi, there expressions which are essentially guard/action
  - differently from Murphi, each action deals with *one variable only*
    - the guard may be defined on any other variable (and it is typically the case)
  - if something is not specified, then it is understood to be non-deterministic
  - indirect specification; also direct specification is allowed, as we will see

NuSMV Input Language: `ASSIGN`

- E.g., in `short.smv` initial states are those in which `state` is `ready` and `request` may be either `Tr` or `Fa`
- Thus, there are 2 initial states $I = \{\langle \texttt{ready}, \texttt{Tr} \rangle, \langle \texttt{ready}, \texttt{Fa} \rangle\}$, which may be represented with $\langle \texttt{ready}, \perp \rangle$
- Also `next(request)` is not specified; before analyzing what does this mean, let us see `next(state)`
- The `case` expression works as follows: the first condition C which is evaluated to true is fired, other true guards possibly following C are ignored

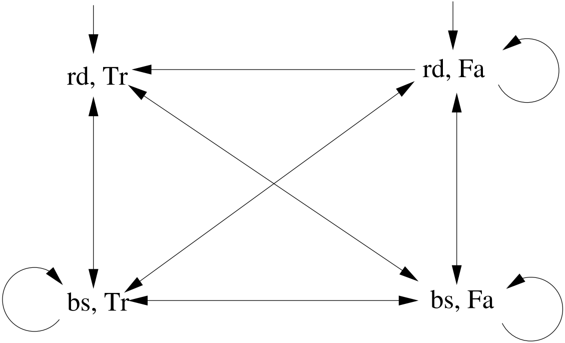
NuSMV Input Language: `ASSIGN`

- This allows to put 1 (i.e., true) as the last guard, representing the "default" case
- NuSMV also checks if a `case` expression is exhaustive in its conditions, as this allows it to assume that R is total
- Note that the last condition on `state` leads to a non-deterministic transition: if the first guard is false, then `state` may take any value between `ready` e `busy`, that is any value in its domain
- In general, any subset of the variable domain may be used

NuSMV Input Language: `ASSIGN`

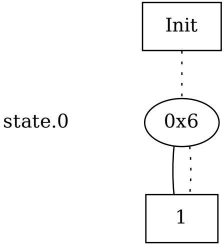
- `request` is completely non-deterministic, as it does not occur in any `next`
- I.e., if other rules tells that the system may go from s to t and $(\texttt{request} = \texttt{Fa}) \in L(t)$, then there exists a transition from s to t' with $(\texttt{request} = \texttt{Tr}) \in L(t')$ and $L(t) \setminus \{(\texttt{request} = \texttt{Fa})\} = L(t') \setminus \{(\texttt{request} = \texttt{Tr})\}$
- Simply stated, if the system may go from s to t and `request` has a value v in t, then the system may also go from s to t' s.t. t and t' only differ in the value of `request`, which is different from v
- By combining all non-determinism in this example, the Kripke structure defined here excludes just two transitions

Automata for `short.smv`: *I* and *R*


rd, Tr
rd, Fa
bs, Tr
bs, Fa

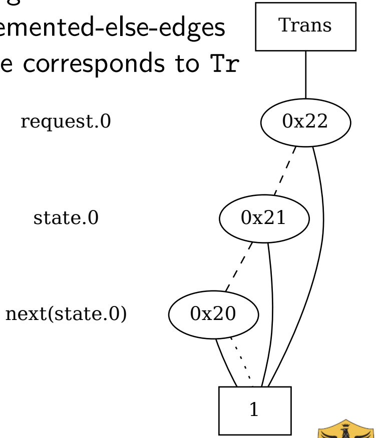
OBDDs for `short.smv`: I

Straight lines are then-edges
Dashed lines are else-edges
Dotted lines are complemented-else-edges


Init
state.0
0x6
1

OBDDs for `short.smv`: *R*

Straight lines are then-edges
Dashed lines are else-edges
Dotted lines are complemented-else-edges
`request.0` “false” edge corresponds to `Tr`


Trans
request.0
0x22
state.0
0x21
next(state.0)
0x20
1

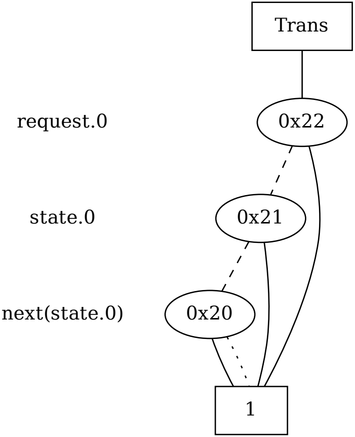
NuSMV Input Language

```
MODULE main
VAR
  request : {Tr, Fa};
  state : {ready, busy};
ASSIGN
  init(state) := ready;
  next(state) := case
                   state = ready & (request = Tr): busy;
                   TRUE : {ready,busy};
                 esac;
SPEC
  AG((request = Tr) -> AF state = busy)
```

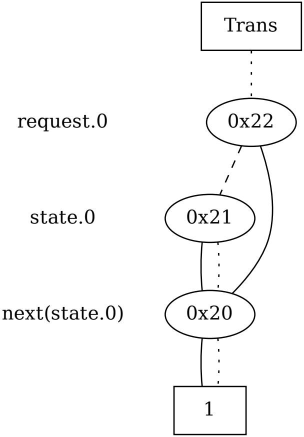
NuSMV Input Language

```
MODULE main
VAR
  request : {Tr, Fa};
  state : {ready, busy};
ASSIGN
  init(state) := ready;
  next(state) := case
                   state = ready & (request = Tr): busy;
                   TRUE : ready;
                 esac;
SPEC
  AG((request = Tr) -> AF state = busy)
```

OBDDs for `short.smv`: R

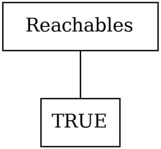

Trans
request.0
0x22
state.0
0x21
next(state.0)
0x20
1

OBDDs for `short.soloready.smv`: R


Trans
request.0
0x22
state.0
0x21
next(state.0)
0x20
1

OBDDs for `short.smv`: Reach

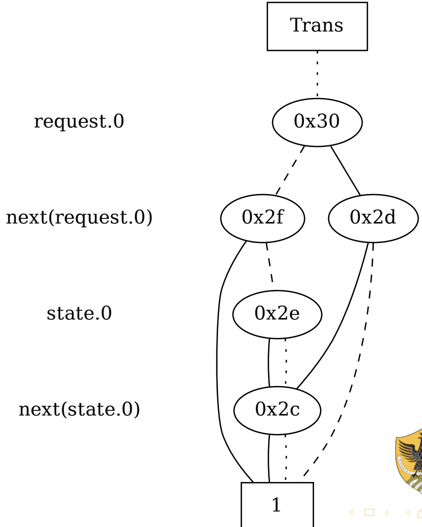
The one for soloready is the same


Reachables
TRUE

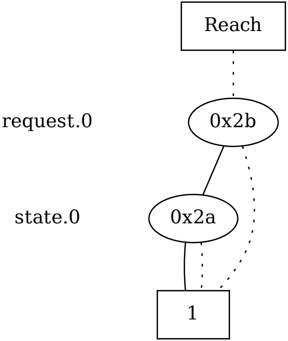
NuSMV Input Language

```
MODULE main
VAR
  request : {Tr, Fa};
  state : {ready, busy};
ASSIGN
  init(state) := ready;
  next(state) := case
                   state = ready & (request = Tr): busy;
                   TRUE : ready;
                 esac;
  next(request) := request;
SPEC
  AG((request = Tr) -> AF state = busy)
```

OBDDs for `short.soloready.req_const.smv`: R


Trans
request.0
0x30
next(request.0)
0x2f
0x2d
state.0
0x2e
next(state.0)
0x2c
1

OBDDs for `short.soloready.req_const.smv`: Reach


Reach
request.0
0x2b
state.0
0x2a
1

OBDDs Pros and Cons

```
MODULE main
VAR
  m1 : 0..15; -- m1.0 is MSB!
  m2 : 0..15;
  m3 : 0..30;
ASSIGN
  next(m3) := m1 + m2;

SPEC
  AG(m3 <= 30);
```

OBDDs Pros and Cons

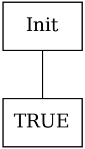
```
MODULE main
VAR
  m1 : 0..15;
  m2 : 0..15;
  m3 : 0..30;
ASSIGN
  next(m3) := case
    m1*m2 <= 30: m1*m2;
    TRUE: m3;
  esac;

SPEC
  AG(m3 <= 30);
```

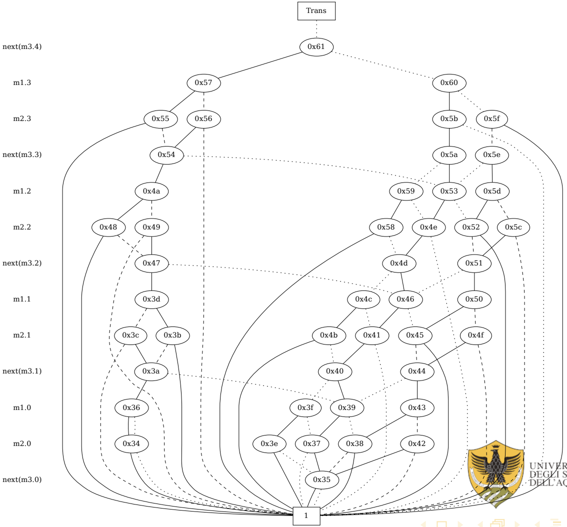
OBDDs for Adder and Multiplier: *I*

This is a set with $16 \cdot 16 \cdot 31 = 7936$ elements

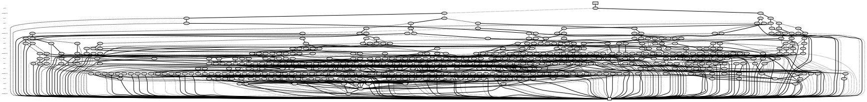
Just one node to represent it...



OBDDs for Adder: *R*


Trans
next(m3.4)
m1.3
m2.3
next(m3.3)
m1.2
m2.2
next(m3.2)
m1.1
m2.1
next(m3.1)
m1.0
m2.0
next(m3.0)
0x61
0x57
0x60
0x55
0x56
0x5b
0x5f
0x54
0x5a
0x5e
0x4a
0x59
0x53
0x5d
0x48
0x49
0x58
0x4e
0x52
0x5c
0x47
0x4d
0x51
0x3d
0x4c
0x46
0x50
0x3c
0x3b
0x4b
0x41
0x45
0x4f
0x3a
0x40
0x44
0x36
0x3f
0x39
0x43
0x34
0x3e
0x37
0x38
0x42
0x35
1

OBDDs for Multiplier: R



OBDDs Pros and Cons

- Number of variables is 13 for both models
  - 4 each for `m1` and `m2`, plus 5 for `m3`
- Number of BDD nodes:
  - adder: 47
  - multiplier: 538

OBDDs Pros and Cons

- No magic: SAT could be solved using OBDDs
  - just represent the instance with an OBDD and check if it is different from 0
  - very roughly speaking: if it were possible to solve it "efficiently" in this way, P=NP...
- Thus, there are boolean functions for which OBDDs representation is exponential, regardless of variable ordering
  - one example is the multiplier seen above
- It is not possible to say if OBDDs will be a good way to represent a problem, before trying it
  - for the adder, it is much more efficient
- Furthermore, finding a variable order in order to minimize the OBDD representation for a given function is an NP-complete problem

OBDDs Pros and Cons

- This also holds for Model Checking in general
- Not possible to say a-priori if a system will fit in the available resources when using a model checker
    - RAM and computation time
- Also, it is not possible to decide which model checker is better
    - explicit (Murphi-or-SPIN like) or symbolic (NuSMV like)?
- However, we are going to see some guidelines
    - as for OBDDs: a good ordering is to interleave present and future variables
    - variable ordering: if OBDDs grow, the model checker can try a different variable ordering

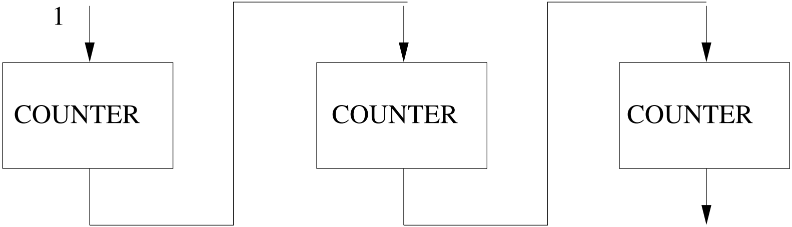
NuSMV Input Language

```
MODULE counter_cell(carry_in)
VAR value : boolean;
ASSIGN
  init(value) := 0;
  next(value) := (value + carry_in) mod 2;
DEFINE carry_out := value & carry_in;

MODULE main
VAR
  bit0 : counter_cell(1);
  bit1 : counter_cell(bit0.carry_out);
  bit2 : counter_cell(bit1.carry_out);

SPEC AG(!bit3.carry_out)
```

Counter Cell


1
COUNTER
COUNTER
COUNTER

NuSMV Input Language

- 2 modules, `main` and `counter_cell`
- Main *instantiates* the module `counter_cell` for 3 times
- This is an hardware-like instantiation: the `main` module contains 3 equal copies of the `counter_cell` module, the only difference being the lines in input
- Note that this means the module `main` will have 3 copies of variable `value`

NuSMV Input Language

- Note that `carry_out` (being inside a `DEFINE` section) is *not* a variable, as it is only a shortcut for the expression it defines
  - i.e., there will not be a corresponding variable in the OBDD
  - and indeed, it is not declared as a variable...
- Hence, `bit0` will always sum 1 to its internal variable, and `bit1` will sum 1 only if `bit0` will generate a carry
- The `main` module defines a counter from 0 to 7

NuSMV Input Language

```
MODULE user(semaphore)
VAR
  state : {idle, entering, critical, exiting};
ASSIGN
  init(state) := idle;
  next(state) :=
    case
      state = idle: entering;
      state = entering & !semaphore: critical;
      state = critical: {critical, exiting};
      state = exiting: idle;
      TRUE : state;
    esac;
```

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NuSMV Input Language

```
next(semaphore) :=
  case
   state = entering: TRUE;
   state = exiting: FALSE;
   TRUE: semaphore;
  esac;
```

NuSMV Input Language

```
MODULE main
VAR
  semaphore : boolean;
  proc1 : process user(semaphore);
  proc2 : process user(semaphore);
ASSIGN
  init(semaphore) := FALSE;

SPEC
  AG(!(proc1.state = critical & proc2.state = critical))

LTLSPEC
  G F proc1.state = critical
```

NuSMV Input Language

- In the previous examples, all variables were evolving at the same time
- There is a global clock as in a synchronous digital circuit: given the current value for all variables in the current clock tick, in the next clock tick all variables may change their variables at the same time (synchronously: hardware parallel execution)
- In this example, instead, instantations are *processes*
- I.e., just one variable at a time may change; other variables are forced to stay fixed
  - this entails that only variables inside the selected process may change
  - other "free" (non-process) variables may change as well, as `sempahore`
  - try this example without processes (and without `RUNNING`)
- No dynamic process spawning as in SPIN: the number of processes is known from the beginning

NuSMV Input Language

- *Synchronous* vs. *asynchronous* systems
- In asynchronous systems, there is essentially one (implicit) additional module, which acts as a scheduler
- This is indeed what the verification algorithm does
- Each process is automatically provided with an additional variable `running` which is true iff that process is currently running

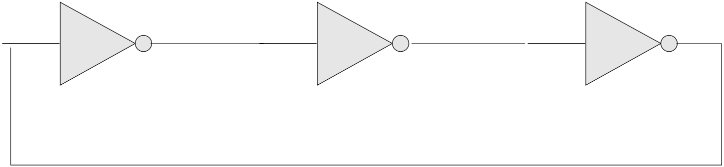
NuSMV Input Language

```
MODULE inverter(input)
VAR
  output : boolean;
ASSIGN
  init(output) := 0;
  next(output) := !input;

MODULE main
VAR
  gate1 : process inverter(gate3.output);
  gate2 : process inverter(gate1.output);
  gate3 : process inverter(gate2.output);

SPEC
  AG(!gate2.output | !gate3.output)
```

Inverter Cell



NuSMV Input Language

Using direct specification it is possible to define non-total transition relations or empty initial states set

```
MODULE inverter(input)
VAR
  output : boolean;
INIT
  output = 0
TRANS
  next(output) = !input
```

NuSMV Input Language

Without processes, is it equivalent?

```
MODULE inverter(input)
VAR
  output : boolean;
ASSIGN
  init(output) := 0;
  next(output) := !input union output;
                -- or {!input, output}

MODULE main
VAR
  gate1 : inverter(gate3.output);
  gate2 : inverter(gate1.output);
  gate3 : inverter(gate2.output);
```

NuSMV As A Tool

- NuSMV is provided with an interactive shell, as there are many tasks it may accomplish (simulation, many verification options); see user manual from chapter 3, especially Figure 3.1 at page 87
- Differently from explicit model checkers, no need to give separate commands to generate a file to be compiled and executed: all is represented as OBDDs, you only have to use them properly
- Executing a non-interactive verification in NuSMV is the same as giving the following list of interactive commands
- 1. `read_model` reads and stores the syntactic structure of the input model
    - no OBDDs here: tree-like structure, but representing the syntactic structure of the input (abstract syntax tree)

NuSMV As A Tool

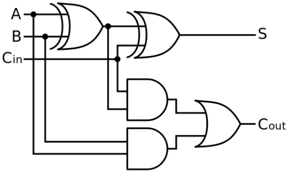
- 2. `flatten_hierarchy` (recursively) brings inside `main` all modules instantiated by `main`
  - very similar to the unfolding we mentioned for Murphi and SPIN: for such explicit model checkers, this was only needed for theoretical purposes, in order to define the Kriepke structure of an input model
  - here, it must be actually performed in the source code of NuSMV, in order to then be able to encode R and I as OBDDs
  - to this aim, there must be only one module, the `main`, containing all variables coming from the modules it instantiates (to be applied recursively)
  - note that, again, this resembles digital circuits, where such a flattening is a natural operation
  - this could entail adding a scheduler module if processes are used

NuSMV As A Tool

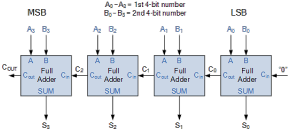
- 3. `encode_variables` for each variable x with domain D s.t. $|D| > 2$, NuSMV defines $x_1 \ldots, x_m$ *boolean* variables with $m = \lfloor \log_2 |D| \rfloor + 1$; it also defines the encoding for constants used in the input models
- 4. `build_flat_model` combines the result of the preceding operations to obtain the flattenized and boolenized syntactic structure which represents the Kriepke structure defined by the input model
- 5. `build_model` from the syntactic structure to OBDDs for R ed I (plus other ones)
- 6. `check_ctlspec` (or `check_ltlspec`, or both, depending on what you have to verify); it starts the actual verification
  - we will be back soon on these last 2 steps

From Syntactic Structure to OBDDs

- How does `build_model` work?
- All operations must be implemented bitwise (*bit-vector*)
  - this means that we have to build the corresponding digital circuit, remember the Digital Systems Design course?
  - if we have to implement a sum between two variables encoded with maximum 4 bits (note that result is on 5 bits):



A
B
Cin
S
Cout



MSB
$A_0 - A_3$ = 1st 4-bit number
$B_0 - B_3$ = 2nd 4-bit number
LSB
A_3 B_3
A_2 B_2
A_1 B_1
A_0 B_0
A B
Full Adder
C_{out}
C_{in}
SUM
C_{OUT}
C_2
C_1
C_0
"0"
S_3
S_2
S_1
S_0

From Syntactic Structure to OBDDs

- Analogously, you can represent other arithmetic operations (subtract, multiply, divide)
- With other simple digital circuits, also equality and ordering can be easily implemented
  - e.g., `next(a) = b + c` is translated in this way:
  - multiple OBDDs are used to sum all bits of `b` and `c`
  - an OBDD B is created which is true iff all variables of `next(a)` are equal to such OBDDs
  - e.g., `next(a) = case a < b:  b + c; TRUE : a` is translated in this way:
  - again we have B as before, plus an OBDD C which is true if `a < b`
  - then, NuSMV computes the OBDD $ITE(C, B, a)$

From a NuSMV Description to KS

- From a NuSMV model $\mathcal{M}$ (defined with the `ASSIGN` section) to the corresponding Kriepke structure $\mathcal{S} = (S, I, R, L)$
    - $V = \langle v_1, \ldots, v_n \rangle$ is the set of variables defined inside the `main` module of $\mathcal{M}$, with domains $\langle D_1, \ldots, D_n \rangle$
        - note that each D_i may be the instantiation of other modules
        - in which case, again, all variables must be considered as *unfolded*
        - that is, if a variable v is the instantiation of a module with k variables, then v counts as k variables instead of one
        - if one of such k variables is another instantiation, this procedure must be recursively repeated
        - NuSMV calls this operation *hierarchy flattening*
        - essentially, it is the same as for records in Murphi
        - simple types are the recursion base step

From a NuSMV Description to KS

- $S = D_1 \times \ldots \times D_n$ (as in Murphi)
- I is defined by looking at `init` predicates
  - $s \in I$ iff, for all variables $v \in V$, $s(v) \in \mathtt{init}(v)$
    - note that, by NuSMV syntax, each $\mathtt{init}(v)$ is actually a set (possibly a singleton)
  - if $\mathtt{init}(v)$ is not specified in $\mathcal{M}$, then any value for v is ok: in this case, formally, if $s \in I$, then also $s' \in I$ being $s'(v') = s(v') \forall v' \neq v$

From a NuSMV Description to KS

- R is defined by looking at `next` predicates
  - we assume all `next` predicates to be defined by the `case` construct (if not, simply assume it is the `case` construct with just one `TRUE` condition)
  - for each (flattened) variable v, we name $g_1(v), \ldots g_{k_v}(v)$ the conditions (guards) of the `case` for `next(`v`)`, and $a_1(v), \ldots a_{k_v}(v)$ the resulting values (actions) of the `case` for `next(`v`)`
  - note that, by NuSMV syntax, each $a_i(v)$ is actually a set (possibly a singleton)
  - $(s, s') \in R$ iff, for all variables $v \in V$, if $g_i(s(v)) \wedge \forall j < i \neg g_j(s(v))$ then $s'(v) \in a_i(v)$
  - that is, s may go in s' iff, for all variables v, if the values of v in s satisfy the guard g_i (and none of the preceding guards for the same variable), then the value of v in s' is one of the values specified by the `case` for guard g_i
  - note that, in doing this, you also have to resolve inputs for modules

From a NuSMV Description to KS

- $AP = \{(v = d) \mid v = v_i \in V \wedge d \in D_i\}$
- $(v = d) \in L(s)$ iff variable v has value d in s
- If, instead, the NuSMV model $\mathcal{M}$ is defined with the `TRANS` section, then
    - $V = \langle v_1, \ldots, v_n \rangle$ is the set of variables as above and $S = D_1 \times \ldots \times D_n$
    - I is defined by looking at `INIT` section
        - $s \in I$ iff, for all variables $v \in V$ and for all `INIT` sections $\mathcal{I}$, $\mathcal{I}(s(v))$ holds
    - R is defined by looking at `TRANS` section
        - $(s, s') \in R$ iff, for all variables $v \in V$ and `TRANS` sections T, $T(s(v), s'(v))$ holds

λ-calculus: Representing Functions

- In a nutshell: using $f(x)$ has some drawbacks
    - you are forced to name a function (f in the example above)
    - it is not always clear if a letter is a parameter or an argument
    - it is not computationally clear what happens for multiple inputs
        - $f(x, y)$: do you have to provide both x, y, otherwise you get an error?
        - as an alternative, you may provide just one argument, and obtain a new function
        - e.g. $f(x, y) = x + y$, we have that $f(x, 4)$ is a function on x

λ-calculus: Representing Functions

- Instead of writing $f(x) = E(x)$, for some expression $E(x)$, we write $\lambda x.E(x)$
  - if you *want*, you can name a function $f(x) = \lambda x.E(x)$
- $\lambda(x, y).x + y$: both arguments must be given, otherwise it is an error
- $\lambda x \lambda y.x + y$: if you provide $x = 4$ only, you get a function $\lambda y.4 + y$
- If an OBDD contains variables $x_1, \ldots, x_n$, then it represent some function $\lambda x_1 \ldots \lambda x_n.\ E(x_1, \ldots, x_n)$

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μ-calculus: Fixpoints

- In a nutshell: we have a set L with an ordering $\leq$
  - $\leq$ could be partial, i.e., not defined on some pair $(l_1, l_2) \in L \times L$
  - $L, \leq$ is a *complete lattice* if any subset $A \subseteq L$ has a greatest lower bound and a least upper bound in L
  - $\sup A = \min\{\xi \in L \mid \forall \alpha \in A.\ \alpha \leq \xi\} \to \sup A \in L$
  - $\inf A = \max\{\xi \in L \mid \forall \alpha \in A.\ \xi \leq \alpha\} \to \inf A \in L$

μ-calculus: Fixpoints

- Let $I = \{0, \dots, 10\}$, then $L = (2^I, \subseteq)$ is a complete lattice
  - e.g., $\{0, 1, 2\} \leq \{0, 1, 2, 3\}$, whilst $\{0, 1, 2\}, \{0, 1, 3\}$ cannot be compared
  - $\sup\{\{0, 1, 2\}, \{0, 1, 3\}\} = \min\{\xi \in 2^I \mid \forall \alpha \in \{0, 1, 2\}, \{0, 1, 3\}.\alpha \subseteq \xi\} = \min\{\{0, 1, 2, 3\}, \dots, I\} = \{0, 1, 2, 3\}$
  - $\inf\{\{0, 1, 2\}, \{0, 1, 3\}\} = \max\{\xi \in 2^I \mid \forall \alpha \in \{0, 1, 2\}, \{0, 1, 3\}.\xi \subseteq \alpha\} = \max\{\{0, 1\}, \dots, \varnothing\} = \{0, 1\}$
- $2^I, \subseteq$ is always a complete lattice, if I is a finite set
  - $\sup J = \cup_{\xi \in J}\, \xi$, $\inf J = \cap_{\xi \in J}\, \xi$
  - at the worst, $\sup J = I$ and $\inf J = \varnothing$

μ-calculus: Fixpoints

- Suppose you have a function $T : L \to L$. An element $\xi \in L$ is a *fixpoint of T* iff $T(\xi) = \xi$
- Given a T, there may be several fixpoints: we are interested in the maximum or the minimum of such fixpoints
  - notation μT and νT
  - where typically T is expressed with a λ notation
  - $\mu T \equiv \xi$ s.t. $T(\xi) = \xi \wedge \forall \rho \in L.T(\rho) = \rho \to \xi \leq \rho$
  - $\nu T \equiv \xi$ s.t. $T(\xi) = \xi \wedge \forall \rho \in L.T(\rho) = \rho \to \rho \leq \xi$

μ-calculus: Fixpoints

- Let again $I = \{0, \ldots, 10\}$
- Let $T : 2^I \to 2^I$ be defined as $T(\xi) = \xi$, or better $T \equiv \lambda\xi.\xi$
  - we have $\mu T = \varnothing$, $\nu T = I$
- Let $T \equiv \lambda\xi.\varnothing$
  - we have $\mu T = \nu T = \varnothing$
- Let $T \equiv \lambda\xi.\ \xi \cup \{10\}$
  - we have $\mu T = \{10\}, \nu T = I$
- Let $T \equiv \lambda\xi.\ \xi \setminus \{10\}$
  - we have $\nu T = \{0, \ldots, 9\}, \mu T = \varnothing$

μ-calculus: Fixpoints

- We define sets by their characteristic function, thus let us rewrite the previous examples
    - thus the ξ in $\lambda\xi$ is a function $\xi : I \to \{0,1\}$
    - it represents a set X, thus $\xi(x) = 1$ iff $x \in X$
- $T \equiv \lambda\xi.\xi$ is ok also if ξ is a characteristic function
- $T \equiv \lambda\xi.\varnothing$ could be rewritten as $T \equiv \lambda\xi.\lambda x.0$
- $T \equiv \lambda\xi.\xi \cup \{10\}$ could be rewritten as $T \equiv \lambda\xi.\lambda x.[x = 10 \to 1] \wedge [x \neq 10 \to \xi(x)]$
    - $\mu T \equiv \lambda x.x = 10, \nu T \equiv \lambda x.1$
- $T \equiv \lambda\xi.\xi \setminus \{10\}$ could be rewritten as $T \equiv \lambda\xi.\lambda x.[x = 10 \to 0] \wedge [x \neq 10 \to \xi(x)]$
    - $\nu T \equiv \lambda x.x \neq 10, \mu T \equiv \lambda x.0$

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μ-calculus: Fixpoints

- We deal with monotonic (i.e., increasing or decreasing) T, thus fixpoints always exists
    - $\xi \leq \rho \rightarrow T(\xi) \leq T(\rho)$, T monotonically increasing
    - $\xi \leq \rho \rightarrow T(\rho) \leq T(\xi)$, T monotonically decreasing
- Previous examples are all monotonic
- By (weak) Knaster-Tarski theorem, $\mu T = \inf\{\xi \mid T(\xi) \leq \xi\}$
    - analogously, $\nu T = \sup\{\xi \mid T(\xi) \geq \xi\}$

μ-calculus: Fixpoints Computation

- Consequence of Knaster-Tarski: computing μT and νT may be done as follows
- For $k \geq 1$, let $T^k(\xi) = T(T^{k-1}(\xi))$, with $T^1 = T$
- For least fixpoints (μT), start with $\varnothing$, and apply T since $T^k(\varnothing) = T^{k-1}(\varnothing)$
  - of course, $\varnothing = \lambda x.0$
- For greatest fixpoints (νT), start with U, and apply T since $T^k(U) = T^{k-1}(U)$
  - of course, $U = \lambda x.1$
- At most, $k = |U|$

Computation of Fixpoints in CTL Model Checking

- Given a KS $\mathcal{S} = (S, I, R, L)$, we want to label states, i.e., to identify subsets of S
  - those for which a given labeling holds
  - labels are CTL/LTL subformulas
- Thus, $L = 2^S$, $\leq$ is $\subseteq$ and $T : 2^S \to 2^S$
  - in the following, $x = x_1, \ldots, x_n$ with $n = \lceil \log |S| \rceil$
  - characteristic functions of subsets of S
  - thus, each subset of S (member of 2^S) is an OBDD
  - hence, a T takes an OBDD and returns another (possibly modified) OBDD
- At most, $k = |S|$
  - usually, much less than that

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μ-calculus: Fixpoints

- The "really interesting" fixpoints are those which are recursively defined
  - typically, basing on some other already defined sets, i.e., characteristic functions
  - e.g., $T \equiv \lambda\xi.\lambda x.f(x) \vee \xi(x)$, where $f : S \to \{0, 1\}$ is known
  - the compactly-written least and greatest fixpoints are $\mu Q.\lambda x.f(x) \vee Q(x)$ and $\nu Q.\lambda x.f(x) \vee Q(x)$
  - e.g., $T \equiv \lambda\xi.\lambda x.f(x) \wedge \xi(x)$
  - e.g., $T \equiv \lambda\xi.\xi(x)$
- By the Knaster-Tarski theorem and the previous reasoning, we may apply the following algorithms
  - least fixpoints μ are computed for increasing T
  - greatest fixpoints ν are computed for decreasing T
  - viceversa are trivial: μT is $\lambda x.0$ for decreasing T and νT is $\lambda x.1$ for increasing T

Computation of Least (Minimum) Fixpoint

```
OBDD lfp(MuFormula $T$) /* $\mu Z.T(Z)$ */
{
  $Q$ = $\lambda x.0$;
  $Q'$ = $T(Q)$;
  /* $T$ clearly says where $Q$ must be replaced */
  /* e.g.: if $\mu Z.\lambda x.f(x) \vee Z(x)$, then
     $Q' = \lambda x.f(x) \vee Q(x)$ */
  while ($Q \neq Q'$) {
    $Q$ = $Q'$;
    $Q'$ = $T(Q)$;
  }
  return $Q$; /* or $Q'$, they are the same... */
}
```

Computation of Greatest (Maximum) Fixpoint

```
OBDD gfp(NuFormula $T$) /* $\nu Z.T(Z)$ */
{
    $Q$ = $\lambda x.1$;
    $Q'$ = $T(Q)$;
    while ($Q \neq Q'$) {
        $Q$ = $Q'$;
        $Q'$ = $T(Q)$;
    }
    return $Q$;
}
```

Symbolic Model Checking of **AG**p

- The idea is to compute the set of reachable states, and check if for all of them p holds
- $\mathrm{Reach} = \mu Z.\, \lambda x.\, [I(x) \vee \exists y :\ (Z(y) \wedge R(y,x))]$
  - of course, we get an OBDD on x as a result
  - recall that x (and y) is a vector of all boolean variables
- $\forall x \in S.\, \mathrm{Reach}(x) \rightarrow p(x)$
  - computationally easier: check that $\mathrm{Reach}(x) \wedge \neg p(x) = 0$
  - otherwise, we have a reachable state for which p does not hold...

Symbolic CTL Model Checking

- All CTL formulas can be reduced to 3: $\mathbf{EX}f$, $f\ \mathbf{EU}\ g$, $\mathbf{EG}f$
  - all other formulas may be reduced to these three, using negation and other boolean combinations
  - with OBDDs, we can do all such things!
- Given OBDDs for f (and g), we compute the OBDD representing $\mathbf{EX}f$, $f\ \mathbf{EU}\ g$, $\mathbf{EG}f$
  - that is, the OBDD for the set $X = \{s \in S \mid \mathcal{S}, s \models \mathbf{EX}f\}$ etc
- Let it be B: then, simply check $\neg B(x) \wedge I(x) = 0$
  - recall that $\mathcal{S} \models \Phi$ iff $\forall s \in I.\ \mathcal{S}, s \models \Phi$
- $\mathbf{EX}f$ does not require a fixpoint computation: it is equivalent to (the OBDD representing) $\lambda x.\ \exists y:\ R(x, y) \wedge f(y)$

Symbolic CTL Model Checking

- For f **EU** g, recall that it is equivalent to the CTL formula $g \vee (f \wedge \mathbf{EX}(f \ \mathbf{EU} \ g))$
- Thus, f **EU** $g = \mu Z.\ \lambda x.\ g(x) \vee (f(x) \wedge \mathbf{EX} Z(x)) = \mu Z.\ \lambda x.\ g(x) \vee (f(x) \wedge (\exists y : \ R(x, y) \wedge Z(y)))$
  - note that $g(x) \vee (f(x) \wedge \mathbf{EX} Z(x))$ is increasing, i.e. for $Z_1 \subseteq Z_2$ we have that $(g(x) \vee (f(x) \wedge \mathbf{EX} Z_1(x)) \rightarrow (g(x) \vee (f(x) \wedge \mathbf{EX} Z_2(x))$
- Analogously: $\mathbf{EG} f = f \wedge \mathbf{EX}(\mathbf{EG} f)$, thus $\mathbf{EG} f = \nu Z.\ \lambda x.\ f(x) \wedge \mathbf{EX} Z(x) = \nu Z.\ \lambda x.\ f(x) \wedge (\exists y : R(x, y) \wedge Z(y))$
  - note that $f(x) \wedge \mathbf{EX} Z(x)$ is decreasing, i.e. for $Z_1 \subseteq Z_2$ we have that $(f(x) \wedge \mathbf{EX} Z_2(x)) \rightarrow (f(x) \wedge \mathbf{EX} Z_1(x))$

CTL Model Checking

```
bool checkCTL(KS $\mathcal{S}$, CTL $\varphi$) {
    let $\mathcal{S} = \langle S, I, R, L \rangle$;
    $B$ = LblSt($\varphi$);
    return $\lambda x.\, I(x) \land \neg B(x) = \lambda x.\, 0$;
}
OBDD LblSt(CTL $\varphi$) { /* also $\mathcal{S} = \langle S, I, R, L \rangle$ */
  if ($\exists p \in AP.\, \varphi = p$) return $\lambda x.\, p(x)$;
  else if ($\varphi = \neg\phi$) return $\lambda x.\, \neg$LblSt($\phi$)($x$);
  else if ($\varphi = \phi_1 \land \phi_2$)
    return $\lambda x.$LblSt($\phi_1$)($x$)$\land$LblSt($\phi_2$)($x$);
  else if ($\varphi = \mathbf{EX}\phi$)
    return $\lambda x.\, \exists y :\ R(x, y)\land$LblSt($\phi$)($y$);
  else if ($\varphi = \mathbf{EG}\phi$)
    return gfp($\nu Z.\, \lambda x.$ LblSt($\phi$)($x$) $\land$ ($\exists y :\ R(x, y) \land Z(y)$)));
  else if ($\varphi = \phi_1$ $\mathbf{EU}$ $\phi_2$)
    return lfp($\mu Z.\, \lambda x.$ LblSt($\phi_2$)($x$)$\lor$
        (LblSt($\phi_1$)($x$) $\land$ ($\exists y :\ R(x, y) \land Z(y)$))));
}
```